

UDC 542-1

Solvability of initial-boundary value problems for a quasi-hydrodynamic system of equations in the Oberbeck-Boussinesq approximation

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Abstract: This paper considers the generalized solvability of an analogue of the first initial-boundary value problem for a quasi-hydrodynamic system of equations in the Oberbeck-Boussinesq approximation for a weakly compressible fluid. The system consists of an elliptic equation for the pressure, a parabolic system for the fluid velocity vector, and a parabolic equation for the temperature. The stationary part of the system is not uniformly elliptic, which complicates obtaining a priori estimates and proving the existence of a solution. The main result of the paper is an existence theorem for generalized solutions. The proof is based on the Galerkin method, choosing a special basis, and the obtained a priori estimates.

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1. Introduction

In recent years, video monitoring and surveillance systems have been widely used in traffic management. For example, Istanbul Traffic Management Company (ISBAK) have started to use more than 500 cameras for traffic monitoring. Extracting useful information such as traffic density and vehicle types from these camera systems has become a hassle due to the high number of cameras in use. Manual analysis of these camera systems is now unapplicable. Development of intelligent systems that extract traffic density and vehicle classification information from traffic surveillance systems is crucial in traffic management.

It is important to know the traffic density of the roads real time especially in mega cities for signal control and effective traffic management. Time estimation of reaching from one location to another and recommendation of different route alternatives using real time traffic density information are very valuable for mega city residents. In addition, vehicle classification (big: truck, middle: van, or small: car) is also important for traffic control centers. For example, the effects of banning big vehicles from a road can be analyzed using vehicle classification information in a simulation program. This paper presents an automatic traffic density estimation and vehicle classification method for traffic surveillance system using neural networks.

Several other vehicle detectors such as loop, radar, infrared, ultrasonic, and microwave detectors exist in the literature. These sensors are expensive with limited capacity and involve installation, maintenance, and implementation difficulties. For example, loop sensor might need maintenance due to road ground deformation or metal barrier near the road might prevent effective detection using radar sensors [1]. In recent years, video processing techniques have attracted researchers for vehicle detection [2-7].

Detection of moving objects including vehicle, human, etc. in video can be achieved in three main approaches: Temporal difference, optical flow, and background subtraction. In temporal difference, the image difference of two consecutive image frames are obtained [12-18]. However, this approach has some limitations such as visual homogeneity requirement and its effectiveness depends on the speeds of moving objects [2]. Optical flow method was developed to obtain effective background modification, which bases on the detection of intensity changes [2]. However, illumination change due to weather or sun-light reflections decreases its effectiveness. It is also computationally inefficient [2]. The third method, background subtraction, is the mostly seen method in the literature for effective motion tracking and moving object identification [2, 4, 6, 9, 10, 11]. In background subtraction, background can be static, in which a fixed background is obtained beforehand and used in the entire process; or dynamic, in which background is dynamically updated with changing external effects like weather. Static background may not be effective in most applications, many methods include dynamic background subtraction. In [19], the background is detected dynamically by using dynamic threshold selection method. In [22], land mark based method and BS&Edge method are used to remove the shadow from the scene.

Different classification techniques have been employed after the moving objects are detected in order to identify the moving object. In [4], support vector machines is used to identify if the detected moving object is a vehicle or not. Support vector machine is a two class classification method and requires modification for multi class classification. The vehicles are detected using mathematical modeling in [21]. The expected parameters of a moving vehicle is mathematically modeled using the position of the camera, vehicle, and sun; it is compared with the values obtained from the video. However, this model requires very sensitive calibration of the camera and it works for cases with short distance between camera and vehicles. The traffic videos used in Istanbul do not satisfy these needs. In [20], rule based reasoning is used for vehicle detection, in which the results highly depend on the rules decided by humans.

Neural networks have been widely used in traffic control [23, 24, 25]. In [23], the traffic incident detection model using neural networks has been developed using traffic magnetic sensors. Intelligent agent systems have been using in [24] in order to control the traffic. Hybrid computational intelligent techniques and fuzzy neural networks has been applied in multi agents in order to control the traffic signals. They have reduced the average waiting time in the traffic. In [25], traffic flow prediction is achieved using time delay based neural networks. In this paper, we will use neural networks for identification of vehicles and traffic density by processing the traffic videos.

In most vehicle detection methods in the literature, only the detection of vehicles in frames of the given video is emphasized. However, further analysis is needed in order to obtain the useful information for traffic management such as real time intensity of roads and number of vehicle types passing these roads. This paper presents vehicle classification and road intensity calculation methods using neural networks. Section 2 discusses the modeling background, section 3 gives the traffic density estimation algorithm and results obtained from traffic videos in Istanbul. Section 4 concludes the paper.

2. Modeling Background

Our model consists of three submodels: Moving Object Detector (MOD), Vehicle Identifier (VI), Traffic Density Calculator (TDC) as shown in Fig.1. In MOD, moving objects are detected using background estimation method. In VI, the vehicles are detected and identified as small, middle, and big using neural network model. In TDC, the traffic density is calculated using the identified vehicle information in successive frames. The details of these submodels are explained in following subsections.

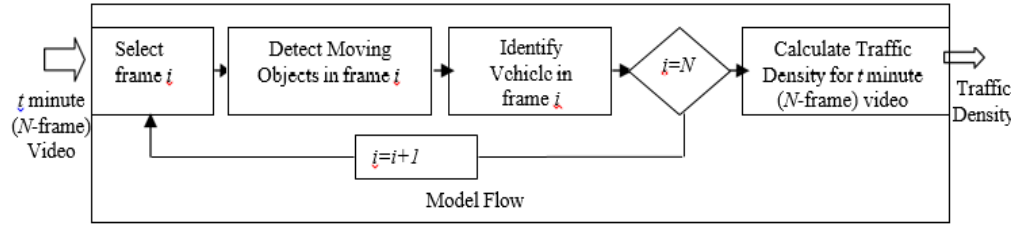


Figure 1: Model Flow

2.1 Moving Object Detector (MOD)

Moving object detection is applied to each frame distinctly and performed as shown in Fig. 3. In the first step, the background of the video is subtracted from the current image and the a threshold is applied to the difference matrix. Then, the moving object is detected by analyzing the difference matrix between background and the current frame image. Lastly, the background is updated with the current frame for the following frames.

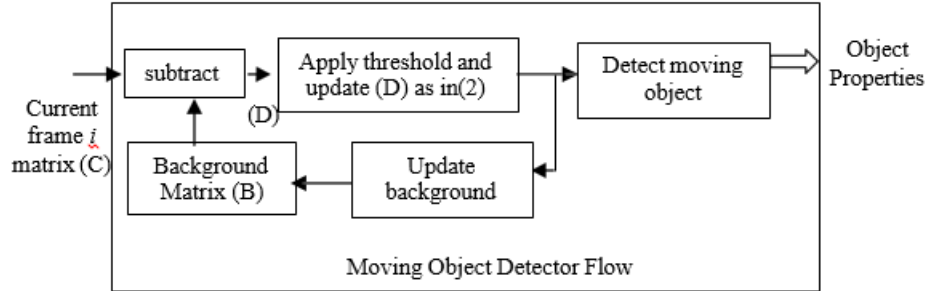


Figure 2: Moving Object Detector Flow

In the first step, the background is subtracted from the current frame. The difference matrix is applied to a threshold. The gray levels greater and lower than the threshold is

$$D_{i,j} = C_{i,j} - B_{i,j} \quad (1)$$

D: Difference matrix with n rows and m Columns

C: Current frame matrix with n rows and m Columns

B: Background matrix with n rows and m Columns

$$D_{i,j} = \begin{cases} 1 & \text{if } D_{i,j} > th \\ 0 & \text{if } D_{i,j} \leq th \end{cases} \quad (2)$$

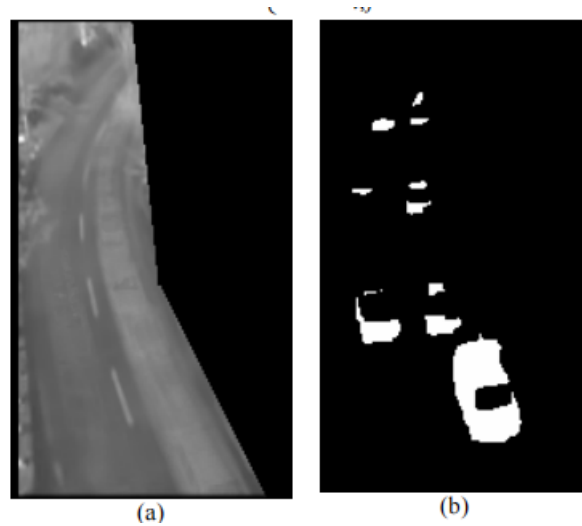


Figure 3: a) Dynamic background b) Difference between the current frame and background

Once the binary matrix showing the difference between current frame and background is obtained, this matrix is analyzed in order to detect the moving object. The biggest area of ones within this matrix is detected. For example there are three objects represented with ones in the matrix given in Fig.4. In order to classify the moving objects, as much information as possible should be extracted. For this purpose, smallest ellipse and rectangle that covers the given object is considered and 14 properties are identified for each object. These properties are given in Fig. 5.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	1	0
0	1	1	1	1	0	1	0	0	1	1
0	1	1	1	1	0	1	0	0	1	1
0	0	0	0	0	0	1	1	0	1	1
0	0	0	0	0	0	1	0	0	1	1

Figure 4. Example of Difference Matrix

As mentioned earlier, the static background (e.g., the image of the road without any vehicle is passing on the road) may be used. However, this approach works only for cases where external factors such as weather, illumination (day, night or time within a day) effects are the same with the static background. Thus, dynamic background calculation methods are common in the literature. In the dynamic background calculation, the background is updated for each frame. In our algorithm, the pixel gray levels are stored as time series data and the median value of the time series data is updated based upon the new gray level in the current frame.

1	P1n	Number of pixels covered for the objects. Number of ones and zeros surrounded with these ones
2	P2n	Number of covering ones of the objects
3,4	cx,cy	Center of the area
5,6,7,8	E1,E2,E3,E4	The coordinates of four edges of the smallest rectangle that can cover the object
9	dx	The distance between smallest and highest indices on x-axis
10	dy	The distance between smallest and highest indices on y-axis
11	O	Angle between the object axis with the x-axis
12	R	Ratio of number of ones to the number of pixels within the smallest rectangle that cover the object
13	Ec	Eccentricity of the smallest ellipse that cover the object
14	di	Diameter of the smallest ellipse that cover the object

Figure 5. Properties to be used to classify a vehicle.

2.2 Vehicle Identifier (VI)

In this work feed-forward Neural Network is used. Neural Networks (NN) have been used for decades for solving problems such as classification, clustering, and function approximation. NNs inspired from biological brains consisting of millions of interconnected neurons. NN calculates an output by processing the inputs through neurons in input, hidden, and output layers. There exist several types of NN methods.

Our Neural Network model consists of 14 input and 4 output layers. Input layers are the 14 object properties identified in MOD and shown in Fig. 5. Output layers are binary valued nodes and each represent a vehicle type (i.e., big, small, and medium vehicle and not a vehicle). Only one output node can be one at a time. In other words, these output nodes compete each other in order to represent the given input. Only one hidden layer is used, which includes 25 nodes. It is optimized using trial-error method. Fig. 6 represents the structure of the neural network model.

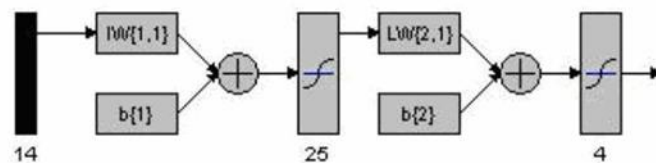


Figure 6 Neural Network Model

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Vi is calculated based on
AllVech=0
For (k=1 to N) // N is the last frame
  For (j=1 to Detk) // Detk is the number vehicles detected in frame k
    AllVech=AllVech+1
    Center[AllVech]=c[j]

Vi = 0
Isclse=false;
while(is AllVech empty)
  select vehicle from AllVech
  [x y] = get the coordinates of selected vehicle
  remove the selected vehicle from AllVech
  for(go back 1 to H number of frames)
    [tempx tempy] =get the coordinates of vehicles in each frame
    if(is tempx and tempy close enough to x and y)
      Isclse = true;
  if(not Isclse)
    Isclse=false;
    Vi = Vi +1
  Store the selected vehicle for future comparisons

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Fig. 7. Vehicle counting algorithm

2.3 Traffic Density Calculator (TDC)

In the first two submodels (MOD and VI), the frames are processed individually. In this submodel, all video frames for a given time period is processed together in order to calculate the number of vehicles that passed through the road for the given time period. Succusive frames represent the scene of the road milliseconds after each other. Thus, the same vehicles will be seen in successive frames. In this submodel, number of vehicles that passed the road for the given time period is counted using the location of the vehicle in successive frames. The traffic density is calculated as the number of vehicles over the time as shown in (3). The number of vehicles is calculated according to the algorithm given in Fig. 7.

$$density_i = \frac{V_i}{T} \quad (3)$$

density_{*i*}: Traffic density of vehicle type *i*

V_i: Number of vehicle type *i* that passed the road in time period *T*

T: Time period

3. Model Application to Real Videos

The presented method is applied to video obtained from one of the traffic cameras used Istanbul traffic management company. One scene of the video is given in Fig. 8. In the selected video, there are three road parallel to each other. Each road consists of two lanes. The left road, which is the most crowded one, is selected in our application. Our model is applied to 1000 frames, which lasts 100 seconds (10 frames in a second).



Figure 8. One scene of the traffic video used in model

Since we are interested in only one road, the rest of the matrix is cut out and the background is calculated only for the selected road as in Fig. 9.

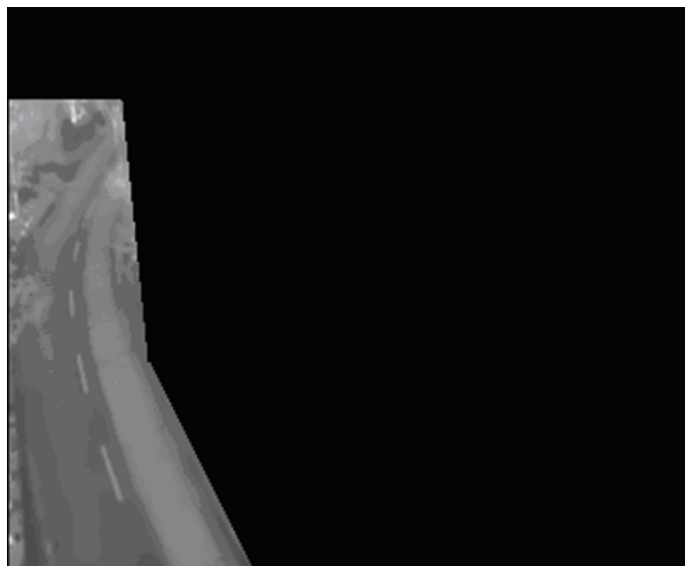


Figure 9. The background image



Fig 10. The difference matrix

The difference matrix is used to identify the vehicle nominees. 14 properties shown in Fig. 5. is used as input for NN model. NN model identifies the vehicles with classification accuracy of 98.9878. After the vehicles in frames are identified, traffic intensity is calculated. In 100 seconds, 68 vehicles has passed from the road. Number of vehicles counted by our algorithm is also 68. In other words, traffic intensity is calculated without any error. However, the classification of vehicle types is performed with some error. Some of the vehicles have been misclassified. The results are given in Table 1. The video is uploaded to the our website at <http://www.fatih.edu.tr/~fcamci/proje/isbak/test.html> for future studies.

Table 1: Classification Results

	Total Vehicles	Small (Cars)	Medium (Van)	Big (Bus)	Unclassified
Real	68	58	9	1	0
Found	68	48	15	1	4
Correctly Identified	68	64			4
Accuracy	100%	94%			6%

4. Conclusion

Automatic traffic density estimation and vehicle classification through video processing and artificial systems are important for traffic management companies especially in mega cities. Traditional traffic density estimation methods such as radars, loop sensors, ultrasonic waves etc. have some limitations. In this paper, automatic traffic density estimation and vehicle classification method using neural network is presented. The presented method is applied to real traffic videos used in Istanbul Traffic Management Company (ISBAK). The results are promising and presented in the paper.

REFERENCES

1. M. Alyürük, *Information Technologies Chief at Istanbul Traffic Management Company (ISBAK), Interview on June 2008*
2. X. Ji, Z. Wei, Y. Feng, *Effective vehicle detection techniques for traffic surveillance systems*, *J. Vis. Commun. Image R.* 17, 647-658, 2006
3. A. Lozano, G. Manfredi, L. Nieddu, *An algorithm for the recognition of levels of congestion in road traffic problems*, *Math. Comput. Simul.*, doi:10.1016/j.matcom.2007.06.08
4. J. Zhou, D. Gao, D. Zhang, *Moving vehicle detection for automatic traffic monitoring*, *IEEE Trans. Vehic. Tech.*, 56(1), 51-59, 2007
5. X. Niu, *A semi-automatic framework for highway extraction and vehicle detection based on a geometric deformable model*, *ISPRS J. Photog. Remot. Sens.* 61 170-186, 2006
6. W. Zhang, X. Z. Fang, X. Yang, *Moving vehicles segmentation based on Bayesian framework for Gaussian motion model*, *Patt. Recog. Lett.* 27, 956-967, 2006
7. X. Li, Z. Q. Liu, K. M. Leung, *Detection of vehicles from traffic scenes using fuzzy integrals*, *Patt. Recog.* 35, 967-980, 2002
8. V. Kastrinaki, M. Zervakis, K. Kalaitzakis, *A survey of video processing techniques for traffic applications*, *Image Vis. Comput.* 21, 359-381, 2003
9. J. B. Kim, H. J. Kim, *Efficient region-based motion segmentation for a video monitoring systems*. *Pattern Recognit. Lett.* 24, 113-128, 2003
10. J. K. Cai, Q. i Aggarwal, *Tracking human motion in structured environments using a distributed camera system*, *IEEE Trans. Pattern Anal. Mach. Intell.* 21 (12), 1241-1247, 1999
11. D. Murray, A. Basu, *Motion tracking with tracking with an active camera*, *IEEE Trans. Pattern Anal. Mach. Intell.* 16 (5), 449-459, 1994
12. M. T. Lo'pez, Ferna'ndez-Caballero, A., Ferna'ndez, M. A., Mira, J., & Delgado,
13. E. (in press). *Dynamic visual attention model in image sequences*. *Image and Vision Computing*.
14. M. T. Lo'pez, Ferna'ndez-Caballero, A., Ferna'ndez, M. A., Mira, J., & Delgado,
15. E. *Visual surveillance by dynamic visual attention method*. *Pattern Recognition*, 39(11), 2194-2211, 2006
16. M. T. Lo'pez, Ferna'ndez-Caballero, A., Mira, J., Delgado, A. E., & Ferna'ndez,
17. M. A. *Algorithmic lateral inhibition method in dynamic and selective visual attention task: Application to moving objects detection and labelling*. *Expert Systems with Applications*, 31(3), 570-594, 2006
18. M. Dubuisson, & A. Jain, *Contour extraction of moving objects in complex outdoor scenes*. *International Journal of Computer Vision*, 14, 83-105, 1995
19. S. Gil & T. Punt, *Non-linear multiresolution relaxation for alerting*. In *Proceedings of the European conference on circuit theory and design*, 1639-1644, 1993
20. M. Teal, & T. Ellis, *Spatial-temporal reasoning on object motion*. In *Proceedings of the British machine vision conference*, 465-474, BMVA Press, 1996
21. R. Waterfall, & K. Dickinson, *Image processing applied to traffic*. *Traffic Engineering Control*, 25, 60-67, 1984
22. M. Fathy and M. Y. Siyal, *A window-based image processing technique for quantitative and qualitative analysis of road traffic parameters*, *IEEE Trans. Veh. Technol.*, 47, (4), 1342-1349, 1998.
23. R. Cucchiara, M. Piccardi, and P. Mello, *Image analysis and rule-based reasoning for a traffic monitoring system*, *IEEE Trans. Intell. Transp. Syst.*, 1,(2), 119-130, 2000

- 24.** Z. Zhu and G. Xu, *VISATRAM: A real-time vision system for automatic traffic monitoring*, *Image Vis. Comput.*, 18(10), 781– 794, 2000
- 25.** M. Yu and Y. D. Kim, *Vision based vehicle detection and traffic parameter extraction*, " *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, E84A,(6),1461–1470, 2001.
- 26.** Nadhir Messai, Philippe Thomas, Dimitri Lefebvre and Abdellah El Moudni, *Neural networks for local monitoring of traffic magnetic sensors* *Control Engineering Practice*, Volume 13, Issue 1, January 2005, Pages 67-80
- 27.** Srinivasan, D., Choy, M.C. and Cheu, R.L., *Neural Networks for Real-Time Traffic Signal Control*, *ITS(7)*, No. 3, September 2006, pp. 261-272.
- 28.** Baher Abdulhai, Himanshu Porwal and Will Recker *Short Term Freeway Traffic Flow Prediction Using Genetically-Optimized Time-Delay-Based Neural Networks*