Математические заметки СВФУ
Апрель–июнь 2026 г. Том 33, № 2

UDC 539-1

Some classes of degenerate quasilinear equations with Riemann-Liouville derivatives and a sectorial pair of operators

A. S. Avilovich , V. E. Fedorov , A. O. Sagimbayeva , D. N. Karpenko

Abstract: The existence and uniqueness of a classical solution to the Showalter-Sidorov problem are considered for certain classes of quasilinear equations in Banach spaces with fractional Riemann-Liouville derivatives. Equations with a degenerate operator at the highest Riemann-Liouville derivative are investigated in the case of a sectorial pair of operators from the linear part of the equation, i.e., when this pair generates a degenerate resolving family of operators analytic in the sector, and under the assumption that the nonlinear operator is independent of the elements of the degeneracy subspace. Theorems on the local and global unique solvability of the Showalter-Sidorov problem for the equation under study are proved. The abstract results obtained are used in the study of initial-boundary value problems for a class of systems of partial differential equations of fractional order with respect to time.

DOI: 10.5281/zenodo.20629543

Keywords: Quasilinear equations; Showalter-Sidorov problem; Riemann-Liouville derivatives; Banach spaces.

1. Introduction

The fixed-point theory of contractive mappings has occupied a central role in mod-ern analysis since the seminal work of Banach [1]. The Banach contraction principle not only guarantees existence and uniqueness of fixed points in complete metric spaces but also ensures convergence of the Picard iteration. Because of its fundamental impor-tance, numerous authors have introduced refinements and generalizations. Among them are the nonlinear contractions of Meir and Keeler [2], the generalized contractive con-ditions of C´ iric´ [3], and the Hardy–Rogers-type inequalities [4]. Comprehensive treat-ments appear in Goebel and Kirk [5], in Khamsi and Kirk [6], in Rus's monograph on generalized contractions [7], and in the detailed study of Lipschitzian-type mappings by Agarwal, O'Regan, and Sahu [8]. More recently, fixed-point theory has been further en-riched by investigations of nonexpansive and multivalued mappings in generalized metric settings. In particular, Dewangan et al. [9] studied multivalued nonexpansive mappings in uniformly convex Banach spaces, highlighting ongoing developments beyond classical contractive frameworks.

This work was supported by the Ministry of National Education, Professional Training, Higher Education and Scientific Research (MENFPESRS), the Digital Development Agency (DDA) of Morocco (Nos. Alkhawarizmi/2020/23), and the National Scientific and Technical Research Centre (CNRST) under the ≪PhD-Associate Scholarship - PASS≫ program.

In a different direction, Edelstein [10] established that strict local contraction suffices to guarantee the uniqueness of fixed points in compact metric spaces, highlighting the importance of geometric features beyond the classical Lipschitz constant. More recently, Petrov [11] introduced a novel contraction condition defined through the perimeter of triangles formed by three iterates of a mapping. His results show that three-point geometric configurations may shrink even when no uniform pairwise contraction exists, thereby opening a new geometric perspective within fixed-point theory.

Another influential development is Burton's concept of large contractions [12], where contractivity is not uniform on the entire space but becomes effective whenever the distance between points is sufficiently large. Burton proved that such mappings still possess a unique fixed point provided one orbit remains bounded, and as noted by Smart [13], this result is closely related to the longstanding question of whether every shrinking selfmap of the unit ball necessarily has a fixed point—Burton's theorem provides an affirmative answer under the assumption of large contractivity. Further advances were obtained by Dehici, Mesmouli, and Karapinar [14], who studied large Kannan-type contractions and demonstrated that, on bounded or compact metric spaces, the orbit-boundedness assump-tion can be considerably weakened or even removed. Additional developments on large contractions include applications to fractional differential equations [15], the characteriza-tion of necessary and sufficient conditions for large contractivity [16], and extended forms of large contractions developed in [17].

Motivated simultaneously by Petrov's geometric method and Burton's nonuniform contraction idea, we introduce in this paper the notion of a large triangle–perimeter contraction. This condition requires strict local reduction of triangle perimeters together with a quantitative contraction property that becomes effective when the smallest side of the triangle is bounded below. Our definition thus blends three-point geometric information with a nonuniform contraction mechanism. We show that this class of mappings properly extends both Petrov's perimeter contractions and Burton's large contractions.

This paper is organized as follows. In Section 2, we recall the basic notions of tri-angle perimeters, Petrov's perimeter contractions, and Burton's large contractions, to-gether with several auxiliary properties used later. In Section 3, we introduce the no-tion of a large triangle–perimeter contraction and establish key preliminary observations. Section 4 contains our main fixed-point theorem, together with a detailed proof based on a perimeter-decay argument adapted from Burton's method. In Section 5, we present continuous and discrete examples showing that our class properly extends Petrov's uniform perimeter contractions. This paper concludes with remarks and potential directions for further research.

2. Preliminaries

Throughout this paper, (X, d) denotes a metric space with |X| ≥ 3. For three distinct points a, b, c ∈ X, we denote by

$$P(a, b, c) := d(a, b) + d(b, c) + d(a, c)$$

the perimeter of the triangle with vertices a, b, c. This quantity is the geometric object whose behavior under iteration of a mapping will be central to our analysis.

Petrov introduced a geometric contraction condition based on the perimeters of triangles rather than pairwise distances.

Definition 1 (Petrov [11]). A mapping T : X → X is called a triangle–perimeter contraction if there exists a constant α ∈ [0, 1) such that

$$P(Tx, Ty, Tz) \leq \alpha P(x, y, z),$$

for all pairwise distinct points x, y, z ∈ X.

Theorem 1 (Petrov's Fixed-Point Theorem [11]). Let (X, d) be a complete metric space. If T: X → X is a triangle–perimeter contraction, then T has a unique fixed point, and for every x0 ∈ X, the sequence {Tnx0} converges to that fixed point.

A central idea due to Burton is that contractivity need not be uniform; it may become effective only when distances are sufficiently large.

Definition 2 (Burton [12]). Let (X, d) be a complete metric space. A mapping T : X → X is a large contraction if:

1. d(Tx, Ty) < d(x, y) for all x ≠ y, and
2. for every ε > 0, there exists a constant δ(ε) ∈ (0, 1) such that

$$x, y \in X, d(x, y) \geq \varepsilon \quad \Rightarrow \quad d(Tx, Ty) \leq \delta(\varepsilon)\, d(x, y).$$

Theorem 2 (Burton's Fixed-Point Theorem [12]). Let (X, d) be a complete metric space and let T : X → X be a large contraction. If there exist x0 ∈ X and L > 0 such that

$$d(x_0, T^n x_0) \leq L, \forall n \geq 1,$$

then T has a unique fixed point in X.

This result connects with a classical question raised by Smart [13]: Does every shrinking map of the unit ball necessarily possess a fixed point? Burton showed that the answer is affirmative provided the map is a large contraction.

Additional Notation and Useful Properties

- For each x_0 ∈ X, the orbit of x0 under T is

$$\mathrm{O}(x_0) := \{x_0, Tx_0, T^2x_0, \ldots\}.$$

- A sequence {xn} in X is Cauchy if

$$\forall\, \varepsilon > 0\ \exists N \in \mathrm{N} : m, n \geq N \Rightarrow d(x_m, x_n) < \varepsilon.$$

- For any distinct points x, y, z ∈ X, the triangle perimeter satisfies

$$\max\{d(x, y), d(y, z), d(x, z)\} \leq P(x, y, z) \leq 3 \max\{d(x, y), d(y, z), d(x, z)\}.$$

Consequently, P(x, y, z) → 0 if and only if all three pairwise distances tend to zero.

- If T is locally strictly contractive on triangle perimeters, then

$$P(x, y, z) = 0 \Rightarrow x = y = z.$$

Thus, perimeter contractivity rules out periodicity of order ≥ 3.

- If $x_n \to x_* \in X$ and $Tx_n \to y_* \in X$, then by continuity of P and the contractive hypotheses, one obtains $Tx_* = y_*$, a fact used in proving fixedness.

These notions and properties serve as the foundation for the definition and analysis of large triangle–perimeter contractions, where local perimeter reduction is combined with nonuniform contraction depending on the minimal side of the triangle.

3. Main Results

We now present our principal fixed-point theorem.

Definition 3 (Large Triangle–Perimeter Contraction). Let (X, d) be a metric space with $|X| \geq 3$. A mapping $T : X \to X$ is a large triangle–perimeter contraction if:

(i) For all pairwise distinct x, y, z, $P(Tx, Ty, Tz) < P(x, y, z)$.

(ii) For every $\varepsilon > 0$, there exists $\delta(\varepsilon) < 1$ such that whenever

$$\max\{d(x, y), d(y, z), d(x, z)\} \geq \varepsilon,$$

then

$$P(Tx, Ty, Tz) \leq \delta(\varepsilon)P(x, y, z).$$

Lemma 1. *Let $(y_0, \ldots, y_r)$ be a finite sequence in a metric space (X, d), with $r \geq 1$, such that*

$$\sum_{j=0}^{r-1} d(y_j, y_{j+1}) \geq \varepsilon_0,$$

for some $\varepsilon_0 > 0$. Then, there exists an index $t \in \{0, \ldots, r-1\}$ such that

$$d(y_t, y_{t+1}) \geq \frac{\varepsilon_0}{r}.$$

Proof. If $d(y_j, y_{j+1}) < \frac{\varepsilon_0}{r}$ for all $j = 0, \ldots, r-1$, then

$$\sum_{j=0}^{r-1} d(y_j, y_{j+1}) < \sum_{j=0}^{r-1} \frac{\varepsilon_0}{r} = r\frac{\varepsilon_0}{r} = \varepsilon_0,$$

which contradicts the hypothesis. Hence, there must exist t with

$$d(y_t, y_{t+1}) \geq \frac{\varepsilon_0}{r}.$$

Theorem 3. Let (X, d) be a complete metric space with $|X| \geq 3$, and let $T: X \to X$ be a large triangle–perimeter contraction in the sense of Definition 3. Assume that there exist $x0 \in X$ and $L > 0$ such that

$$d(x_0, T^n x_0) \leq L \qquad \text{for all } n \geq 1. \qquad (1)$$

Then, T has a unique fixed point in X.

Proof. Define the orbit $(x_n)_{n\geq 0}$ by $x_{n+1} := Tx_n$, with x_0 as in (1). For each $n \geq 0$, set

$$P_n := P(x_n, x_{n+1}, x_{n+2}) = d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_n).$$

Step 1: Strict decrease in the triangle perimeters. If for some n we have xn = Txn, then xn is a fixed point and we are done. Thus, assume that xn ≠ Txn for all n. In particular, the points xn, xn+1, xn+2 are pairwise distinct for all n, so Pn > 0 and condition (i) of Definition 3 yields

$$P_{n+1} = P(Tx_n, Tx_{n+1}, Tx_{n+2}) < P(x_n, x_{n+1}, x_{n+2}) = P_n.$$

Hence, (Pn) is strictly decreasing and bounded below by 0, so there exists ℓ ≥ 0 such that (Pn) converges monotonically to ℓ, that is,

$$P_{n+1} < P_n \quad \text{for all } n \geq 0 \text{ and } \quad \lim_{n\to\infty} P_n = \ell.$$

From the boundedness of the orbit (1), we also have

$$x_n \in \overline{B(x_0, L)} \quad \forall n,$$

so there exists a constant $C > 0$ such that $P_n \leq C$ for all $n \geq 0$.

Step 2: No cycles. Suppose there exist integers $i < j$ such that $x_i = x_j$. Applying T twice, we obtain

$$x_{i+1} = Tx_i = Tx_j = x_{j+1}, \qquad x_{i+2} = T^2x_i = T^2x_j = x_{j+2}.$$

Thus

$$P_i = P(x_i, x_{i+1}, x_{i+2}) = P(x_j, x_{j+1}, x_{j+2}) = P_j,$$

which contradicts the strict decrease Pj < Pi for j > i. Therefore, as long as no fixed point occurs along the orbit, all x_n are distinct.

Step 3: The orbit is Cauchy. We show that (x_n) must be a Cauchy sequence. Assume, for a contradiction, that (x_n) is not Cauchy. Then, there exists ε0 > 0 such that for every N, one can find indices m > n ≥ N with

$$d(x_m, x_n) \geq \varepsilon_0. \tag{2}$$

Fix such an $\varepsilon_0 > 0$. By condition (ii) in Definition 3, there is a constant

$$\delta_0 := \delta(\varepsilon_0) \in (0, 1)$$

such that for any triple (u, v, w) with

$$\max\{d(u, v), d(v, w), d(u, w)\} \geq \varepsilon_0,$$

so (3) applies and yields

$$P(Tx_{n_k}, Tx_{m_k}, Tx_0) \leq \delta_0\, P(x_{n_k}, x_{m_k}, x_0).$$

Iterating this inequality, we obtain for every integer $j \geq 1$,

$$P(T^j x_{n_k}, T^j x_{m_k}, T^j x_0) \;\leq\; \delta_0^j\, P(x_{n_k}, x_{m_k}, x_0). \tag{5}$$

In terms of the orbit (x_n), this reads

$$P(x_{n_k+j}, x_{m_k+j}, x_j) \;\leq\; \delta_0^j\, P(x_{n_k}, x_{m_k}, x_0) \;\leq\; \delta_0^j C,$$

since all perimeters are bounded by C.

Because $d(a, b) \leq P(a, b, c)$ for any a, b, $c \in X$, we deduce

$$d(x_{n_k+j}, x_{m_k+j}) \;\leq\; P(x_{n_k+j}, x_{m_k+j}, x_j) \;\leq\; \delta_0^j C \qquad \forall j \geq 1. \tag{6}$$

Fix $\eta := \varepsilon_0/2 > 0$. Choose J large enough so that

$$\delta_0^J C < \eta = \frac{\varepsilon_0}{2}.$$

By (6), we then have

$$d(x_{n_k+J}, x_{m_k+J}) < \frac{\varepsilon_0}{2} \qquad \text{for every } k.$$

Now, observe that the tail sequence (xn+J)n≥0 is also not Cauchy (because a tail of a nonCauchy sequence is nonCauchy). Hence, by applying (2) to the shifted sequence (xn+J), there exist indices

$$m'_k > n'_k \geq 0 \text{ with } n'_k \to \infty$$

Such that

$$d(x_{m'_k+J}, x_{n'_k+J}) \;\geq\; \varepsilon_0 \quad \text{for all } k.$$

But this contradicts the estimate

$$d(x_{n_k+J}, x_{m_k+J}) < \frac{\varepsilon_0}{2}$$

obtained above (we may relabel the pairs so that they match). Therefore, our assumption that (x_n) is not Cauchy must be false. We conclude that (x_n) is a Cauchy sequence.

Step 4: Existence of a fixed point. Since (X, d) is complete and (x_n) is Cauchy, there

exists $x* \in X$ such that $x_n \to x*$ as $n \to \infty$. We now show that $x*$ is a fixed point of T. From the definition of P_n and the strict decrease, we have $P_n \to 0$. In particular,

$$d(x_n, x_{n+1}) \leq P_n \longrightarrow 0.$$

Thus

$$d(x_n, Tx_n) = d(x_n, x_{n+1}) \to 0.$$

Using the triangle inequality,

$$d(x*, Tx*) \le d(x*, x_n) + d(x_n, Tx_n) + d(Tx_n, Tx*).$$

The first term tends to 0 because $x_n \to x*$, the second tends to 0 because $d(x_n, Tx_n) \to 0$, and the last term tends to 0 since the orbit x_n is bounded, and the large contraction condition implies that T is continuous along this orbit. Hence, $d(x*, Tx*) = 0$, so $Tx* = x*$ and $x*$ is a fixed point.

Step 5: Uniqueness of the fixed point. Suppose $y* \in X$ is another fixed point, $Ty* = y*$, with $y* \neq x*$. Consider the triple $(x*, y*, x_n)$. For n large, these three points are pairwise distinct, and at least one of the distances $d(x*, y*)$, $d(x*, x_n)$, $d(y*, x_n)$ is bounded below by $d(x*, y*) > 0$; hence, for all sufficiently large n, we have

$$\max\{d(x*, y*), d(y*, x_n), d(x*, x_n)\} \ge d(x*, y*).$$

Applying condition (ii) with $\varepsilon = d(x*, y*)$ gives a $\delta \in (0, 1)$ such that

$$P(Tx*, Ty*, Tx_n) \le \delta P(x*, y*, x_n).$$

But $Tx* = x*$ and $Ty* = y*$, so

$$P(x*, y*, x_{n+1}) \le \delta P(x*, y*, x_n).$$

Iterating this inequality yields

$$P(x*, y*, x_n) \le \delta^n P(x*, y*, x_0) \longrightarrow 0 \quad \text{as } n \to \infty.$$

Since $x_n \to x*$, it follows that

$$P(x*, y*, x_n) \longrightarrow d(x*, y*),$$

hence $d(x*, y*) = 0$, a contradiction. Thus, the fixed point is unique.

4. Examples

In this section, we present several examples showing that the class of large triangle–perimeter contractions developed in this paper is strictly broader than Petrov's uniformly contractive triangle maps. In particular, a mapping may fail to satisfy any global coefficient α < 1 for

$$P(Tx, Ty, Tz) \le \alpha P(x, y, z),$$

yet still satisfy local strict perimeter decay together with a uniform perimeter contraction whenever the triangle is "non-small" in the sense that

$$\max\{d(x, y), d(y, z), d(x, z)\} \ge \varepsilon.$$

These examples demonstrate that both continuous and discrete dynamical systems fall naturally within the framework of large triangle–perimeter contractions, even when they clearly violate Petrov's uniform condition.

Example 1. *Let* $X = [0, 1]$ *with the metric* $d(x, y) = |x - y|$ *and define*

$$T(x) = \frac{x}{1 + x}.$$

We show that T is not a uniform triangle–perimeter contraction in the sense of Petrov, but it is a large triangle–perimeter contraction in the sense of Definition 3.

1. T is not a Petrov contraction. Petrov requires the existence of a constant α < 1 such that

$$P(Tx, Ty, Tz) \le \alpha\, P(x, y, z) \qquad (\forall x, y, z \text{ distinct}).$$

Using triples of the form (ε, 2ε, 3ε), a direct calculation shows

$$\frac{P(Tx, Ty, Tz)}{P(x, y, z)} = \frac{1}{4}\left(\frac{2}{3\varepsilon+1} + \frac{1}{2\varepsilon+1} + \frac{1}{\varepsilon+1}\right) \longrightarrow 1 \quad as \quad \varepsilon \to 0.$$

Thus, no uniform α < 1 works.

2. T is a large triangle–perimeter contraction.

(a) Local strict perimeter reduction. Since

$$T'(x) = \frac{1}{(1+x)^2} < 1$$

We have

$$|T(x) - T(y)| < |x - y| \qquad (x \neq y),$$

and hence P(Tx, Ty, Tz) < P(x, y, z) for all distinct triples.

(b) Uniform contraction for large triangles. Assume

$$\max\{d(x, y), d(y, z), d(x, z)\} \ge \varepsilon > 0.$$

For any pair (x, y), we have by the mean value theorem:

$$|T(x) - T(y)| = \frac{|x - y|}{(1+\xi)^2},$$

with ξ ∈ [x, y]. Since at least one of the differences is ≥ε, we obtain

$$|T(x) - T(y)| \le \frac{1}{(1+\varepsilon)^2}|x - y|.$$

Thus

$$P(Tx, Ty, Tz) \le \frac{1}{(1+\varepsilon)^2} P(x, y, z).$$

Hence,

$$\delta(\varepsilon) = 1/2$$

verifies condition (ii).

Therefore, T is a large triangle–perimeter contraction but not a uniform Petrov contraction.

Example 2. Let X = N0 = {0, 1, 2, . . . } with d(m, n) = |m − n| and define

$$P(Tx, Ty, Tz) \leq \frac{1}{2} P(x, y, z).$$

$$T(n) = \left\lfloor \frac{n}{2} \right\rfloor.$$

We show that T fails Petrov's uniform condition but satisfies the large triangle–perimeter contraction condition.

1. Failure of Petrov's condition. Petrov requires a uniform α < 1 such that

$$P(Tx, Ty, Tz) \leq \alpha\, P(x, y, z).$$

For triples (2k, 2k + 1, 2k + 2), we have P(Tx, Ty, Tz) = 2 and P(x, y, z) = 4, so the ratio is 1/2. But for triples (0, 1, N) with N → ∞,

$$P(x, y, z) = N + 1 + N = 2N + 1, \quad P(Tx, Ty, Tz) = P(0, 0, \lfloor N/2 \rfloor) = 2\lfloor N/2 \rfloor,$$

and the ratio tends to 1. Hence, no uniform α < 1 works.

2. Local strict perimeter reduction. For all m ≠ n,

$$|T(m) - T(n)| \leq \frac{1}{2}|m - n|,$$

and therefore P(Tx, Ty, Tz) < P(x, y, z) for every distinct triple.

3. Uniform contraction for large triangles. If

$$\max\{d(x, y), d(y, z), d(x, z)\} \geq \varepsilon > 0,$$

then since

$$|T(u) - T(v)| \leq \tfrac{1}{2}|u - v|$$

for all u, v, we obtain

Hence, $\delta(\varepsilon)$ = 1/2 *works for every* $\varepsilon > 0$.

Consequently, $T(n) = \lfloor n/2 \rfloor$ *is a large triangle–perimeter contraction although it is not a uniform Petrov contraction.*

These examples confirm that the large triangle–perimeter condition captures a sub-stantially broader class of nonlinear mappings than Petrov's uniform perimeter contraction. In particular, local strict perimeter decay combined with uniform contractivity for suf-ficiently "large" triangles (as expressed through the maximal side-length condition in Definition 3) allows both continuous and discrete systems to satisfy our hypotheses even when no global perimeter contraction is possible. This illustrates that our main fixed-point theorem applies naturally to a wider range of dynamical systems and iterative processes.

5. Conclusions and Future Work

In this paper, we introduced the concept of large triangle–perimeter contractions, a class that extends both Petrov's uniform perimeter contractions and Burton's large contractions. By combining strict local perimeter decay with uniform contraction on non-degenerate triangles, we established a Burton-type fixed-point theorem ensuring existence and uniqueness of fixed points for orbit-bounded iterations in complete metric spaces. The examples in Section 4 show that this new framework is strictly broader than Petrov's theory and applies naturally to both continuous and discrete nonlinear mappings.

Several natural extensions arise from our results. One direction is to study large triangle–perimeter contractions in b-metric spaces, where generalized triangle inequalities may lead to new geometric behaviors. Another promising line is the development of Meir–Keeler, Hardy–Rogers, or Reich-type perimeter contractions, which could yield sharper fixed-point criteria. Finally, motivated by recent applications of large contractions in differential and integral equations, it would be interesting to investigate whether the perimeter-based approach introduced here can provide new existence results for nonlinear integral equations, delay equations, or fractional models.

Conflicts of Interest: The authors declare no conflicts of interest.

REFERENCES

1. *Banach, S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. Fundam. Math. 1922, 3, 133–181. [CrossRef]*

2. *Meir, A.; Keeler, E. A theorem on contraction mappings. J. Math. Anal. Appl. 1969, 28, 326–329. [CrossRef]*

3. *C´ iric´, L.B. A generalization of Banach's contraction principle. Proc. Am. Math. Soc. 1974, 45, 267–273. [CrossRef]*

4. *Hardy, G.E.; Rogers, T.D. A generalization of a fixed point theorem of Reich. Can. Math. Bull. 1973, 16, 201–206. [CrossRef]*

5. *Goebel, K.; Kirk, W.A. Topics in Metric Fixed Point Theory; Cambridge University Press: Cambridge, UK, 1990. [CrossRef]*

6. *Khamsi, M.A.; Kirk, W.A. An Introduction to Metric Spaces and Fixed Point Theory; John Wiley & Sons: New York, NY, USA, 2011. [CrossRef]*

7. *Rus, I.A. Generalized Contractions and Applications; Cluj University Press: Cluj-Napoca, Romania, 2001.*

8. *Agarwal, R.P.; O'Regan, D.; Sahu, D.R. Fixed Point Theory for Lipschitzian-Type Mappings with Applications; Springer: New York, NY, USA, 2009. Available online: https://cir.nii.ac.jp/crid/1971149384778339504 (accessed on 4 January 2026).*

9. *Dewangan, K.; Rathour, L.; Mishra, V.N.; Raiz, M. On multi-valued nonexpansive mappings in UCBS. J. Comput. Anal. Appl. 2024, 33, 396–406.*

10. *Edelstein, M. On fixed and periodic points under contractive mappings. J. Lond. Math. Soc. 1962, 37, 74–79. [CrossRef]*

11. *Petrov, E. Fixed point theorem for mappings contracting perimeters of triangles. J. Fixed Point Theory Appl. 2023, 25, 74. [CrossRef]*

12. *Burton, T.A. Integral equations, implicit functions, and fixed points. Proc. Am. Math. Soc. 1996, 124, 2383–2390. [CrossRef]*

13. *Smart, D.R. Fixed Point Theorems; Cambridge University Press: Cambridge, UK, 1980. Available online: https://cir.nii.ac.jp/crid/ 1970586434865391763 (accessed on 4 January 2026).*

14. *Dehici, A.; Mesmouli, M.B.; Karapinar, E. On the fixed points of large-Kannan contraction mappings and applications. Appl. Math.-E-Notes 2019, 19, 535–551. Available online: https://www.math.nthu.edu.tw/~amen/2019/ (accessed on 4 January 2026).*

15. *Mesmouli, M.B.; Akın, E.; Iambor, L.F.; Tunç, O.; Hassan, T.S. On the fixed point theorem for large contraction mappings with applications to delay fractional differential equations. Fractal Fract. 2024, 8, 703. [CrossRef]*

16. *Burton, T.A.; Purnaras, I. Necessary and sufficient conditions for large contractions in fixed point theory. Electron. J. Qual. Theory*

17. *Differ. Equ. 2019, 2019, 1–24. [CrossRef]*

18. *Gülyaz-Özyurt, S. A fixed point theorem for extended large contraction mappings. Results Nonlinear Anal. 2018, 1, 46–48. Available online: https://dergipark.org.tr/en/pub/rna/issue/36561 (accessed on 4 January 2026).*

A. S. Avilovich
Department of Mathematics and Computer Science, University of Oradea, Universitatii nr. 1, 410087 Oradea, Romania

V. E. Fedorov
Department of Mathematics, College of Science, University of Ha'il, Ha'il 2440, Saudi Arabia

A. O. Sagimbayeva
Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

D. N. Karpenko
Mathematics and Data Science Laboratory, Polydisplinary Faculty of Taza, USMBA, Morocco