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The Design and Perceptions of Genetic Decompositions: A case of Mathematics and Physical Science Teachers

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Abstract: Abstract

Exploring how individuals think is a difficult process. We have used written and verbal responses by mathematics and physical science teachers after they were provoked to complete certain tasks. These responses were key to finding out about their thinking. In this paper, we report on a qualitative study which explored the design of genetic decompositions (GD) for some tasks in mathematics and physical science by high school teachers (n=20). They were divided into six groups. Two groups worked on two tasks in physical science and four groups worked on two tasks in mathematics. The genetic decompositions were presented by a group representative to the whole audience. An open-ended questionnaire retrieved data on the teachers' views on the value of genetic decompositions in the teaching and learning of mathematics and physical science. The study was carried out at a University of Technology in South Africa. The data were analysed using APOS theory. Interviews followed these analyses to verify the APOS levels they identified as necessary for students to be successful in achieving correct solutions. Findings emanating from the data analysis indicated that: 1) physical science teachers could justify the action level of a GD but failed to provide sound justification for placing the process and object conceptions in accordance with APOS theory, 2) The physical science teachers failed to identify the mechanisms by which a process or object can be attained, 3) the mathematics teachers' construction of a GD showed that the mental constructions from the PGD3 were similar but the justification for the specific APO mental construction could differ, 4) there is a connection between logistical thinking during learning and the hierarchy of mental constructions in APOS and 5) GD support both teaching and learning by helping them identify learners' challenges, refine instructional strategies, plan learning activities, design tasks aligned with students' mental constructions, and thereby guide learners more efficiently toward higher mental stages while better addressing misconceptions.

Keywords: Actions, process, objects, schema and genetic decompositions.

1. Introduction

The carrying out of mathematical and physical science actions and processes are regarded as human activity. This means that Mathematics and Physical Science (Mathematical Sciences) are used in our daily lives, for example measuring time, distance, mass and volume, and counting money, livestock, etc. Engineers make use of Mathematics and Physical Science in project design (Brijlall, 2017). Engineers require to gain most of their Mathematical Sciences competences at tertiary institutions where they are registered to carry out their initial training to become engineers. Lecturers servicing these students should therefore ensure that these students gain a thorough understanding of their Mathematical Sciences before exiting universities so that they can apply this knowledge in their lifelong careers.

This work was supported by the Ministry of National Education, Professional Training, Higher Education and Scientific Research (MENFPESRS), the Digital Development Agency (DDA) of Morocco (Nos. Alkhawarizmi/2020/23), and the National Scientific and Technical Research Centre (CNRST) under the ≪PhD-Associate Scholarship - PASS≫ program.

The poor performance in the Mathematical Sciences by learners in an international challenge (Abakah & Brijlall 2024a; 2024b). There is a worldwide concern about learners' poor performance in mathematics. In general, Mathematical Sciences are considered as important university subject all over the world. This may be due to the fact that it plays a role in many fields such as artificial intelligence, technology, accounting, and technical subjects. International studies have shown that South Africa has the poorest performance when compared to other middle-income countries and low-income African countries that participated in cross-national assessments of educational achievement, especially in mathematics (Brijlall, 2017). The performance of students in Mathematics is poor when compared to other subjects.

In South Africa, the academic performance of engineering students has become a yardstick for measuring our economic status. At the same time, these academic performances are used as a standard to determine which students qualify to further their studies to higher level Mathematical Sciences modules and post-graduate studies. The major problem is that most tertiary institutions require a high mark in the Mathematical Sciences as an entry requirement for a career in engineering and as a prerequisite for further mathematical engagement. In order to explore the barriers created by learners when working with the problems in Mathematics and Physical Science, the following research questions were conceived: 1. What mental constructions do Mathematics and Physical Science teachers develop when designing genetic decompositions? and 2. What are teachers' perceptions of genetic decomposition? Similar APOS studies were carried out by Brijlall (2017), Brijlall & Maharaj (2017), Brijlall & Ndlazi (2019), Jojo, Maharaj & Brijlall (2013) with engineering students and Bansilal, Brijlall & Trigueros (2017), Brijlall & Maharaj (2011), Ndlovu & Brijlall (2019) with pre-service teachers.

2. Literature Review and Theoretical Framework

Tachie & Chireshe (2013) stated that Mathematics is a prerequisite for learners who wish to pursue a career in higher education. Despite its being an important subject, students continue to perform below expectations in Mathematics and Physics. They found that very few students achieve good marks in prerequisite modules so as to go on with higher-level modules or post-graduate studies. This means that only a small portion of undergraduate students graduate and enter the job market. This study explored the lack of possible mental constructions which lead to unsuccessful application by learners through the lens of their teachers. It is hoped that the findings inform pedagogy so as to improve student performance.

To understand school learners' understanding of concepts in the Mathematical Sciences, we made use of APOS theory in the same manner as done by Abakah & Brijlall (2024a, 2024b). APOS is an acronym for Action – Process – Object – Schema. APOS theory is a framework for research and curriculum development. Tziritas (2011) defines APOS theory as a framework for instructional research in mathematics education. This theory appears within a research framework that has to be conducted in three steps, namely:

- Theoretical analysis of the content to be taught and learned
- Design and implementation of instruction
- Collection and analysis of data

According to Tziritas (2011), these steps are expected to be repeated until satisfactory results regarding students' learning and understanding of the content are obtained. The repetition of steps makes the research process to be cyclical. The continuous analysis of data leads to the collection of more data or to a redesign of the instructional methods (Tziritas, 2011).

Dubinsky and McDonald (2002) explain that the APOS theory begins with the hypothesis that mathematical knowledge is an individual's tendency to deal with perceived mathematical problem situations by constructing mental actions, processes and objects and organising them in schemas to make sense of the situations and solve the problems.

Dubinsky and McDonald (2002) also define the components of APOS theory as follows:

Actions: It is a transformation of objects perceived by the individual as essentially external and as requiring either explicating or from memory, step-by-step instructions on how to perform the operation. An example of this is when students are required to determine if a given square matrix is singular or not (when finding the rank of a matrix). If the student carries out this determination by going along the first row (using minors and cofactors) for every square matrix following a set of computations. This means that the student has learnt a rule and does not inspect the matrix entries to see if there are easier or quicker and less tedious ways to do this.

Process: This is when an action is repeated, and the individual reflects it so that they can think of performing the same kind of action, but no longer with the need for external stimuli. In this case suppose the student wants to determine the singularity of a matrix whilst attempting to determine the rank of a matrix but identifies that the choice of any row or column with entries possessing zero/s will result in less tedious computations then this student has interiorised the procedure for finding out whether a matrix is singular or not.

An object: This is described as what is formed from a process. An individual becomes aware of the process as a totality and realises that transformations can act on it. An illustration of this is when a student can perform an action on the process of conception of a rank. The student can determine the rank of a matrix and the augmented matrix and act on them by comparison and then carry out the procedure of determining the nature of solutions for a system of equations.

A schema: This is an individual's collections of actions, processes, objects and other schemas, which are linked by some general principles to form a framework in the individual's mind that may be brought to bear upon problem situations involving related concepts. In my case I shall speak of a nature of a simultaneous solutions for a system of linear equations schema which will include actions, processes and object conceptions of the various aspects leading to a complete understanding of the rank concept and the nature of simultaneous solution to a system of linear equations.

According to Dubinsky and McDonald (2000), APOS theory can be used directly in the analysis of data by a researcher. It is said that a researcher can compare the successes or failures of students in mathematical tasks with the specific mental constructions they may or may not have made. The example given is that if two learners who agree in their performance up to a very specific mathematical point and then one of students takes a further step while the other cannot, the researcher can explain the differences by pointing to the mental constructions of actions, process, objects and/or schemas that the former learner appears to have made but the other has not.

Another vital phenomenon in APOS theory is the concept of genetic decomposition.

According to Arnon, Cottrill, Dubinsky, Oktaç, Fuentes, Trigueros, Weller, (2014) a genetic decomposition (GD) is a hypothetical model that describes the mental structures and mechanisms that students might need to construct to learn a specific mathematical concept. They further stated that genetic decomposition starts as a hypothesis based on the researcher's experiences in the learning and teaching of concepts, their knowledge of concept, their knowledge of APOS Theory,

APOS theory was also used by Ndlovu and Brijlall (2015) in research in undergraduate mathematics education. The study was carried out using the assumption that understanding the mental constructions that the pre-service teachers make when learning matrix algebra concepts leads to improved instructional methods (Ndlovu & Brijlall, 2015).

Ndlovu and Brijlall (2015) explain that they used APOS theory in their study to describe and analyse pre-service teachers' knowledge constructed of matrix algebra concepts. They say that the APOS theory was used to reveal the nature of the students' mental constructions, not to compare students' performance in matrix algebra concepts. They give examples that illustrate each of the mental constructions mentioned earlier.

Ndlovu & Brijlall (2019) explored the mental constructions displayed by pre-service Mathematics teachers (PMT) when applying Cramer's rule. The aim of their study was to reveal the nature of mental constructions made around the nature of the solution set of equations and the role of parameters in the solution of equations with parametric coefficients. The mental constructions occurred within Action –Process-Object-Schema (APOS) theory. In that study by Ndlovu & Brijlall (2018), data was generated from first year pre-service teachers (n = 31) by means of an activity sheet and interviews. The interviews were used to verify the correct APO mental construction the participants were working at as concluded from the pre-service teachers' responses to tasks from the activity sheet. The findings revealed that most of the PMT displayed procedural understanding of Cramer's rule and of the meaning of the solution of the system of linear equations. This meant the students were operating at the action stage in terms of APOS. Also, it revealed that the lack of construction of related schemata negatively impacted the PMT's attempt to develop the necessary mental constructions. Therefore, the researchers provided a GD for the use of Cramer's rule to aid teacher educators to analyze the mental constructions of students.

In Brijlall (2017), the focus of his study was the causes of poor performance by electrical engineering students in tasks based on the application of the Fourier Transform. The data, which was collected from written responses of students to two tasks, were analysed using APOS theory. Findings emanating from that data analysis indicated that the causes of poor performance emanated from a variety of challenges. Some the contribution causes identified included lack of integration skills and conceptual knowledge of functions. Furthermore, the study found that that different APOS levels of understanding were required for different tasks and at different stages of the tasks.

Brijlall & Ndlazi (2019) explored undergraduate engineering students' understanding of integration. After the analysis, they proposed a GD which is expected to improve the quality of pedagogy in integral calculus. The sample of thirty undergraduate engineering students were taught many different techniques of integration. After that they were confronted with activity sheets with tasks designed in accordance with the ACE cycle. Their written responses, which were used to identify their mental constructions of the integration concepts, were analyzed using APOS theory. Interviews were carried out to clarify the written responses of these engineering students. The discussions and written responses revealed that these students exhibited procedural tendencies in integration, and the students could not define both definite and indefinite integrals. The vital outcome of this research was the formulation of GDs.

3. Methodology

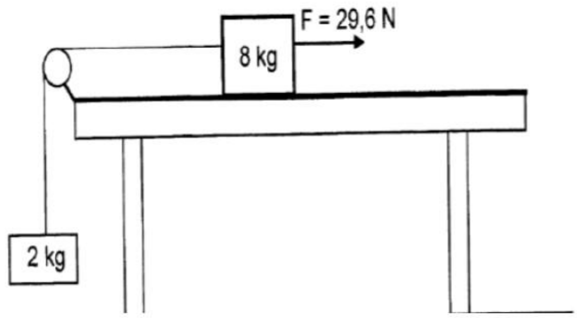
Creswell (2014) defines qualitative research as an approach for exploring and understanding the meaning individuals or groups ascribe to a social or human problem. The process of research involves emerging questions and procedures, data that is collected in the participant's setting, data analysis inductively building from particulars to general themes, and the researcher making interpretations of the meaning of the data (Creswell 2014).

In qualitative research, the main goal is to relate or describe incidents or events of individuals in their natural setting such as an organization, a school, workplace, or home Creswell (2014). That is to say, qualitative research is conducted in the natural setting for which the study was proposed. In this study, the natural setting was the lecture room in which lectures took place.

The focus of our study was the construction of genetic decompositions including mental constructions which should lead to success in solving tasks in Mathematics and Physical Science. The researchers first provided a discourse on APOS theory with a special focus on genetic decompositions with fourteen Mathematics and six Physical Science teachers who teach both Physics and Chemistry. The authors of this study had no control over the choice of the participants as the school principals chose the teachers to attend the workshop. These teachers had varying qualifications and number of years of experience. In another breakout session, the teachers worked in groups. Teachers formed their own groups and chose a group representative. The groups were divided according to the subject they taught in school. There were two groups (each with three participants) of Physical Science teachers and four groups (two with three participants and two with four participants) of Mathematics teachers. Thereafter, the teachers worked with activity sheets which provided tasks for which they had to construct genetic decompositions. We then collected the written responses. The teachers also had to complete a short questionnaire querying their perceptions of genetic decompositions. The mathematics teachers were coded as MT1, MT2, ..., MT14 and the physical science teachers were coded as PT1, PT2, ..., PT6

For the physics teachers, the task (see Figure 1) dealt with the understanding and application of Newton's Laws, specifically Newton 's First Law (Newton i) and newton's Second Law (Newton ii). The task requires recall of Newton ii, identifying which law is applicable for the different state of motion.

An 8kg block, on a rough surface is connected to a 2kg block by a light inextensible string passing over a frictionless pulley, as shown below. The 8kg block moves at a constant speed when pulled by a 29,6 N horizontal force to the right. The frictional force on the 8kg block is 10N.



2.1. State Newtons' second law of motion. (2)
2.2. Draw a free-body diagram for the 8kg block. (5)

2.3 Calculate the tension in the string. (3)
The 29,6 N force is now replaced by 50N.
2.4. Apply Newton's second law to each object and determine:
2.4.1. the acceleration on the 8kg block. (5)
2.4.2. the tension in the string. (2) [17]

Figure 1 - Task P1: Physics problem dealing with Newtons' First and Second Laws

For this task in physics, we propose a GD in Figure 2. For the successful achievement of a solution for task of 2.1 and 2.2 (see Figure 1), the teachers were firstly required to recall the statement of the law. Secondly, the word "rough" is a cue for the individual to identify that the force of kinetic friction was present. These two aspects led us to concluding that the individual was required to have an action conception of Newton's Laws.

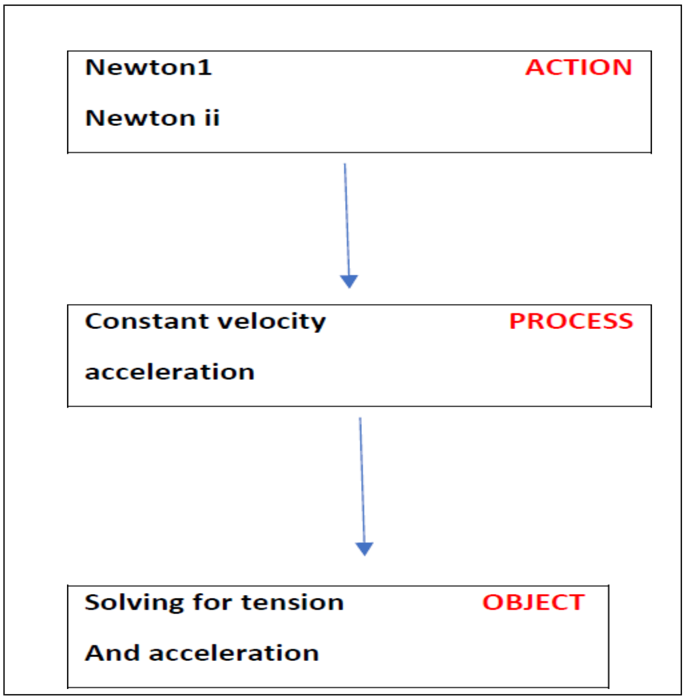


Figure 2: Proposed genetic decomposition for Newtons Laws task (PGD1)

To correctly solve tasks 2.3 and 2.4, the teachers need to have a process conception of Newton's Laws as in task 2.3 the system is in equilibrium as it moves with a constant velocity and so the acceleration is zero and so will the resultant force. In task 2.4, the resultant force is no longer zero, and so the block will accelerate. But to solve the tension and acceleration we identified the mental construction required for successful solution to be an object conception. We say this because the teachers now need to carry out actions on the process conception for Newton's Law. The teachers will be required to isolate each block and apply Newton ii. Two equations will be set up, and a simultaneous solution will lead to the solutions for the magnitudes of the tension and acceleration.

The task for the chemistry teachers is shown in Figure 3. It tested the knowledge and application of concepts of pH, indicators, and titrations involving acids and bases. In Figure 4, we propose a GD for the conceptual understanding of these concepts.

A 200 cm^3 solution of hydrochloric acid has a pH of 1.

8.1 Calculate the volume of water that has to be added to this solution to change the pH to 2. (5)

8.2 Sodium carbonate crystals ($Na_2CO_3.10H_2O$) is used to neutralise the **original** hydrochloric acid (with a pH of 1).

8.2.1 Write down the balanced equation for the neutralisation reaction. (3)

8.2.2 Calculate the mass of sodium carbonate crystals that will be required to neutralise 200 cm^3 of the hydrochloric acid solution. (6)

8.2.3 Choose from the following table the most suitable indicator for the reaction. (1)

Indicator	**pH range in which the colour changes**
Methyl red	4,8 – 6,0
Neutral red	6,8 – 8,0
Chlorophenol red	7,0 – 8,8

8.2.4 Give a reason for the answer to QUESTION 8.2.3. (2)

[17]

Figure 3 - Task C1: Chemistry task on acids and bases.

For task 8.3, a recall of "what an indicator is" was asked. Also, the explanation of the choice of the indicator required the teacher to identify the strength of the acids and bases involved in the titration. This was straight forward recall and so we placed such conceptual understanding as an action (see Figure 4).

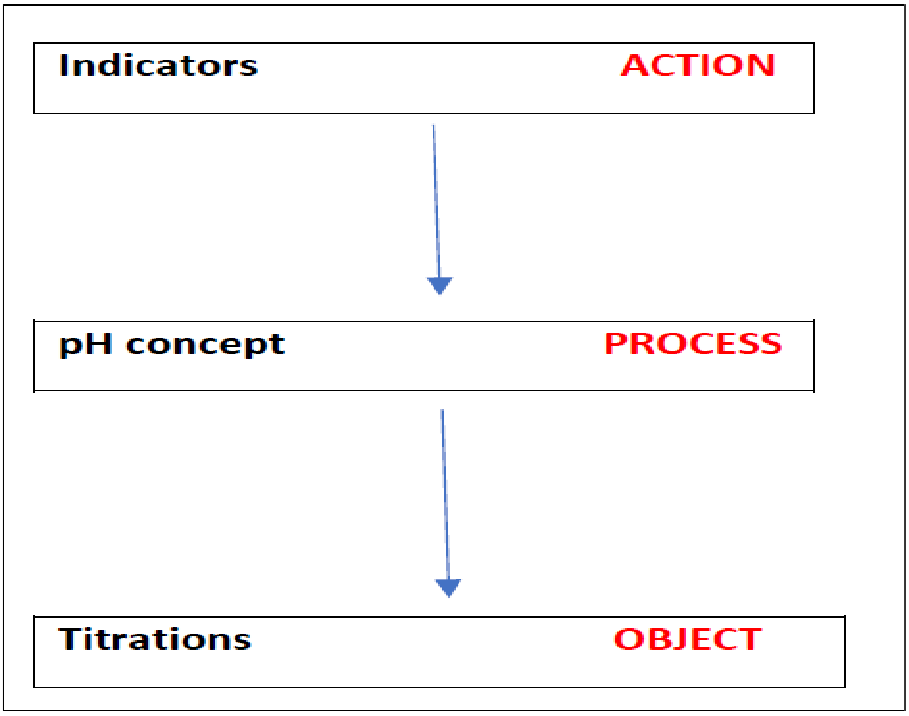


Figure 4 - Proposed genetic decomposition for Chemistry (PGD2)

For task 8.1 in Figure 3, the teachers needed to have a process level of understanding of the concepts of pH and concentration calculations (see Figure 4). The formulae for pH and concentration were required. The teachers needed to reverse the action of using the pH as the pH was given, and the teachers had to find the amount of hydrochloric acid. Then they needed to relate the change in the volume of the solution to change the pH. concentration and hence the new pH. For task 8. 2, The process conception of pH and concentration need to be encapsulated and so an object concept of titrations was required.

For the mathematics teachers, we provided them with a task in financial mathematics. Is task tested concepts of depreciation of assets, investments and loans (see Figure 5).

6.1 How long will it take (answer to the nearest year) for the value of an investment to depreciate with a quarter of its original value? Rate of depreciation is 8,2% p.a. on the reducing balance method. (4)

6.2 Ina wants to travel overseas in 6 years' time. She will need R58 480 to do that. Calculate her monthly payment into a savings account with an interest rate of 9% p.a. compounded monthly if she makes her first payment immediately and her last payment two months before the end of the 6 years. (5)

6.3 Jacob took out a loan of R1 500 000 to buy a house. He will repay the loan with monthly payments over 20 years.
The interest rate is 8% p.a. compounded quarterly.

6.3.1 Showing ALL your calculations and formulae, prove that his monthly instalment will be R12 499,96. (5)

6.3.2 Calculate the outstanding amount after 12 years. (3)

[17]

Figure 5 - Task M1: Financial mathematics task

The GD for Task M1 is illustrated in Figure 6. For task 6.1, the teachers needed to recall the formula for reducing balance depreciation. Thereafter, they had to substitute the relevant values into the formula and by using a calculator to arrive at the answer. This procedure of solving task 6.1 was placed at an action conception of an individual's mental construction.

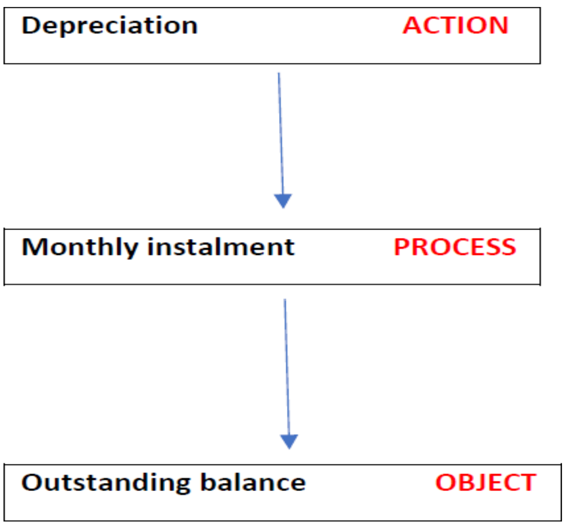
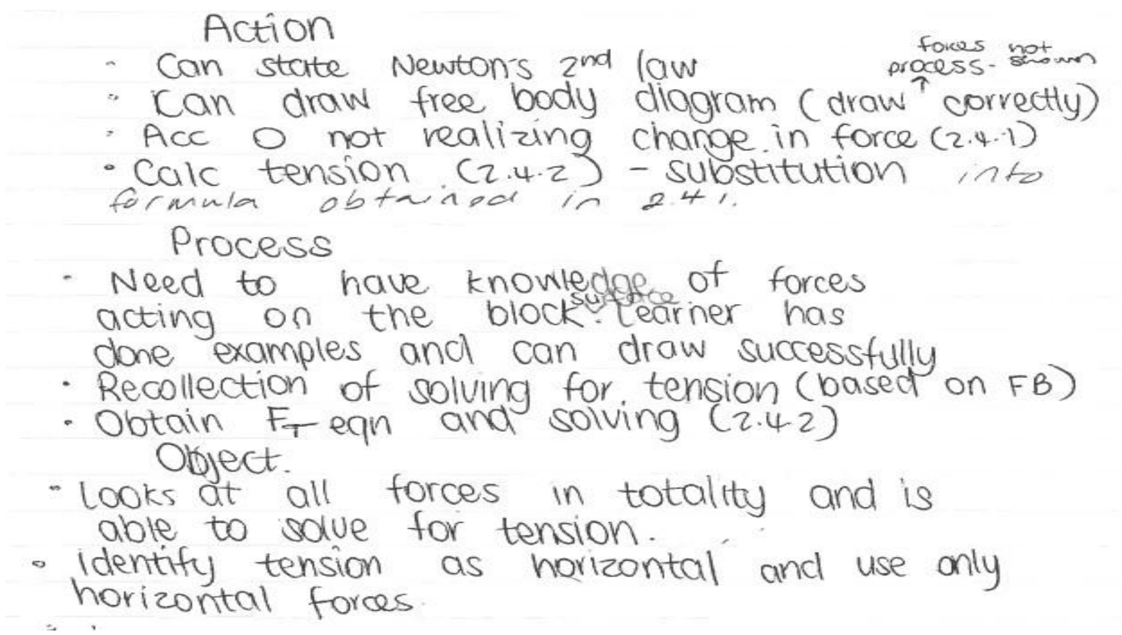


Figure 6: Proposed genetic decomposition for Mathematics (PGD3)

For task 6.2, the teachers needed to interiorise the action conception of the future vale investment into a process. This is due to the deposits being made at irregular times. Since the last payment was made two months before the withdrawal of the investment, two future value formulae are needed to solve the task. For Task 6.3, the process conception of an annuity needed to be encapsulated into a mental object to calculate the balance outstanding on a loan.

4. Data analysis and discussion

Each group representative presented their design of a GD. We present and discuss a group presentation for a task from Physical Science (one for physics and one for chemistry):

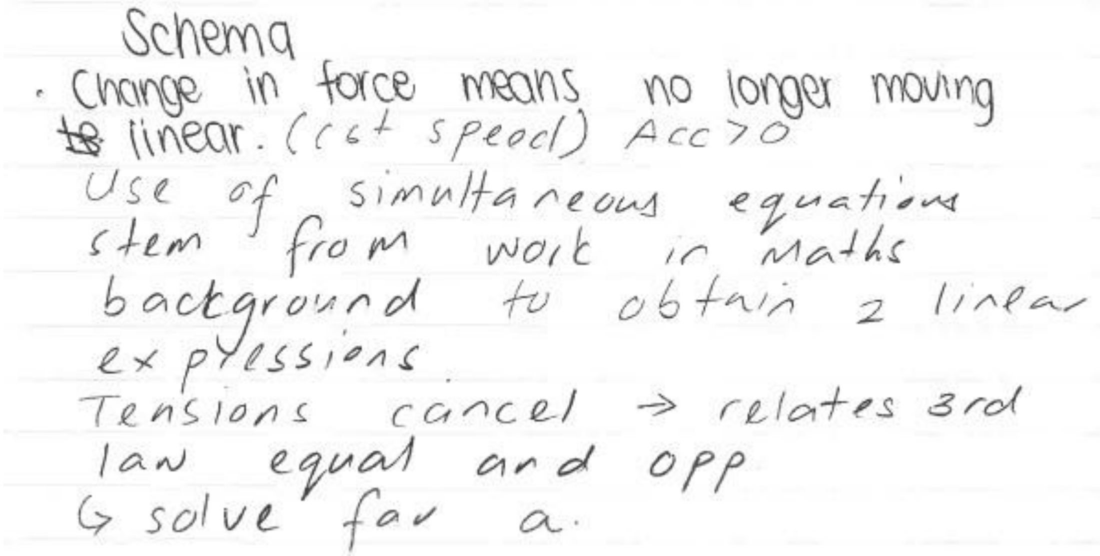

Action
- Can state Newton's 2nd law
- Can draw free body diagram (draw ↑ correctly) forces not shown process.
- Acc 0 not realizing change in force (2.4.1)
- Calc tension (2.4.2) - substitution into formula obtained in 2.4.1.

Process
- Need to have knowledge of forces acting on the block/surface. Learner has done examples and can draw successfully
- Recollection of solving for tension (based on FB)
- Obtain F_T eqn and solving (2.4.2)

Object.
- Looks at all forces in totality and is able to solve for tension.
- Identify tension as horizontal and use only horizontal forces.

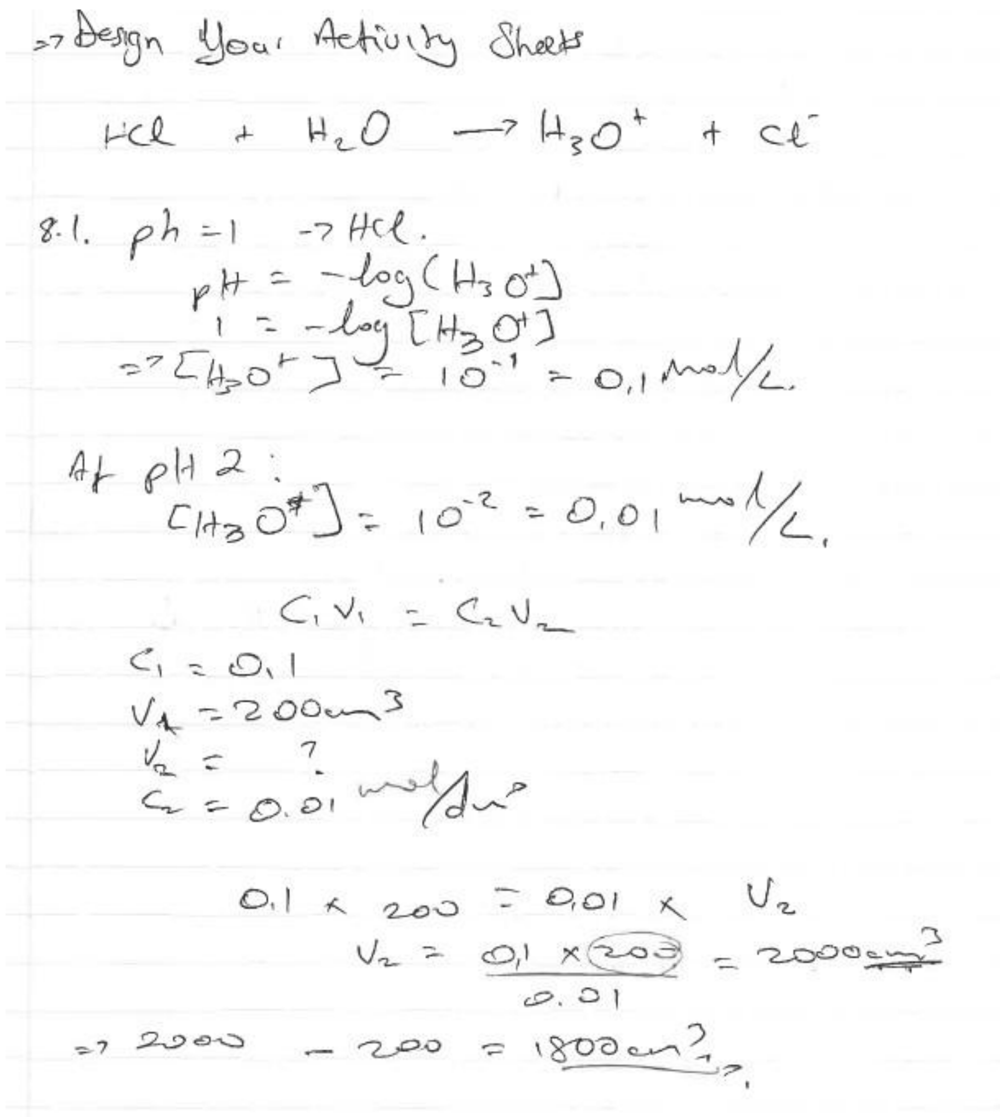
Scan 1: A group's presentation of a GD for physics task

This group's response to tasks 2.1 and 2.2 coincided with the proposed GD presented in Figure 2. This group indicated that stating Newton ii and drawing correctly the free body diagram made these tasks be placed at an action conception. Furthermore, they indicated that substitution meant that calculating the tension was also at an action level. This group focused on the horizontal forces and thought that the use of these horizontal forces be placed at an object level of APOS. This view was different from the proposed GD in that the proposed GD considered the deeper understanding of Newtons Laws. Also, the concept of equilibrium and non-equilibrium (acceleration) were omitted from their GD.



Scan 2: A group's presentation of a GD for physics task

We observe that the mental constructions, for correct solution of task 2.4, are placed by the group (Scan 1) at the APOS levels, as proposed by the GD in Figure 2. However, the reasons offered for such placement of the mental constructions were different from the rationale for the proposed GD in Scan 2.



=> Design Your Activity Sheets

$HCl + H_2O \rightarrow H_3O^+ + Cl^-$

8.1. ph = 1 → HCl.

$pH = -\log(H_3O^+)$

$1 = -\log[H_3O^+]$

$\Rightarrow [H_3O^+] = 10^{-1} = 0{,}1\ mol/L.$

At pH 2:

$[H_3O^+] = 10^{-2} = 0{,}01\ mol/L.$

$C_1V_1 = C_2V_2$

$C_1 = 0{,}1$

$V_1 = 200cm^3$

$V_2 = ?$

$C_2 = 0.01\ mol/dm^3$

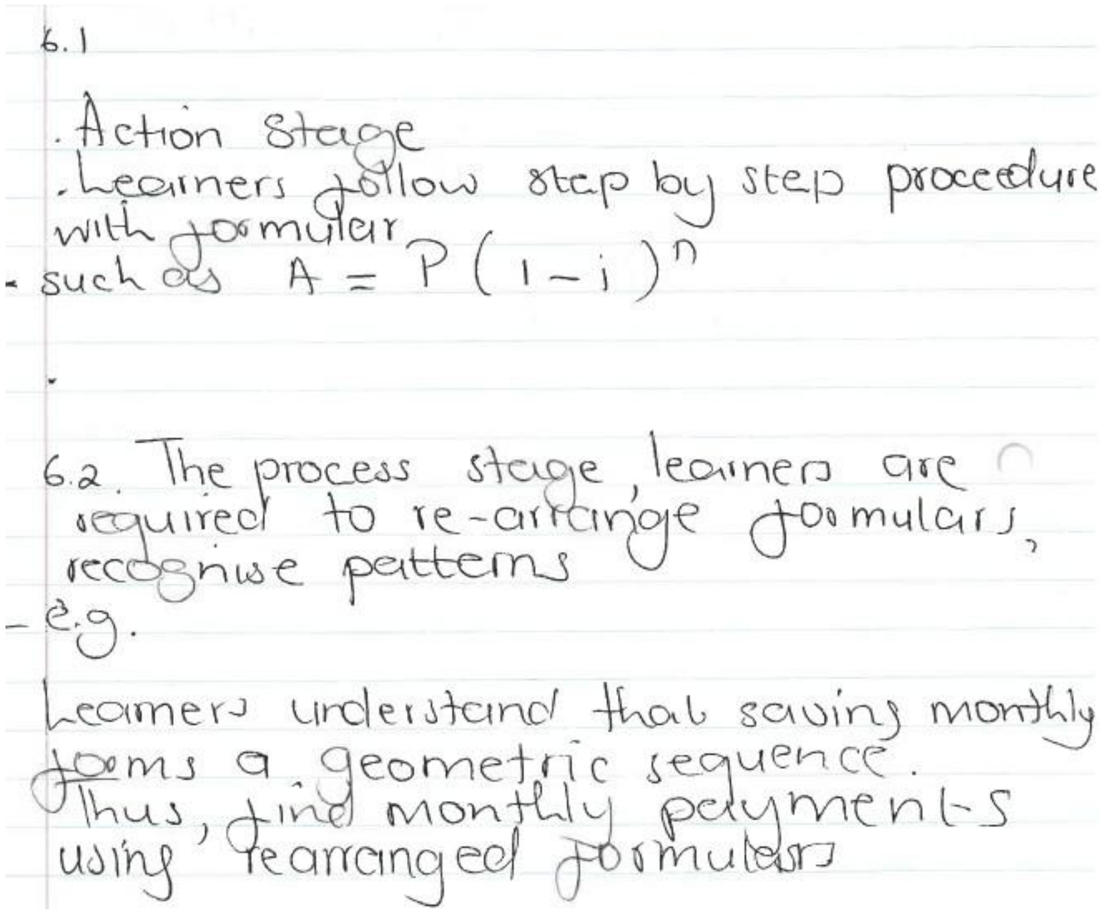
$0{,}1 \times 200 = 0{,}01 \times V_2$

$V_2 = \frac{0{,}1 \times 200}{0.01} = 2000cm^3$

$\Rightarrow 2000 - 200 = 1800cm^3.$

Scan 3: A group's presentation of a GD for chemistry task

We now present a group discussion on the design of a genetic decomposition for a Mathematics task. When comparing the GD provided by the mathematics teachers in Scan 4 with the PGD3 in Figure 6, we observe that the teachers concurred an action conception by an individual to solve task 6.1. This was in keeping with the definitions provided by APOS theory of what an action is.


6.1

. Action Stage
. Learners follow step by step procedure with formular
- such as $A = P(1-i)^n$

6.2. The process stage, learners are required to re-arrange formulars, recognise patterns
- e.g.
Learners understand that saving monthly forms a geometric sequence. Thus, find monthly payments using rearranged formulars

Scan 4: A group's presentation of a GD for mathematics task

For task 6.2, The PGD3 also placed a process conception by an individual for correct solution. We observe that the justification for this differs in both cases. For the PGD3 formulation by the researchers, the reason was that a process conception was needed since the intervals for the instalments were not regular. In the case of the mathematics teachers, they thought that a process conception was needed since a schema for geometric number patterns would assist in attaining a correct solution.

At the end of the session, the teachers completed a short questionnaire asking them for their views on the use of genetic decompositions for pedagogy and for learning. Table 1 summarizes the responses. Favorable refers to teachers who believed that genetic decompositions were beneficial for teaching and learning, while unfavorable refers to those who felt they were not beneficial. Neutral refers to teachers who were unsure whether to accept or reject the use of genetic decompositions.

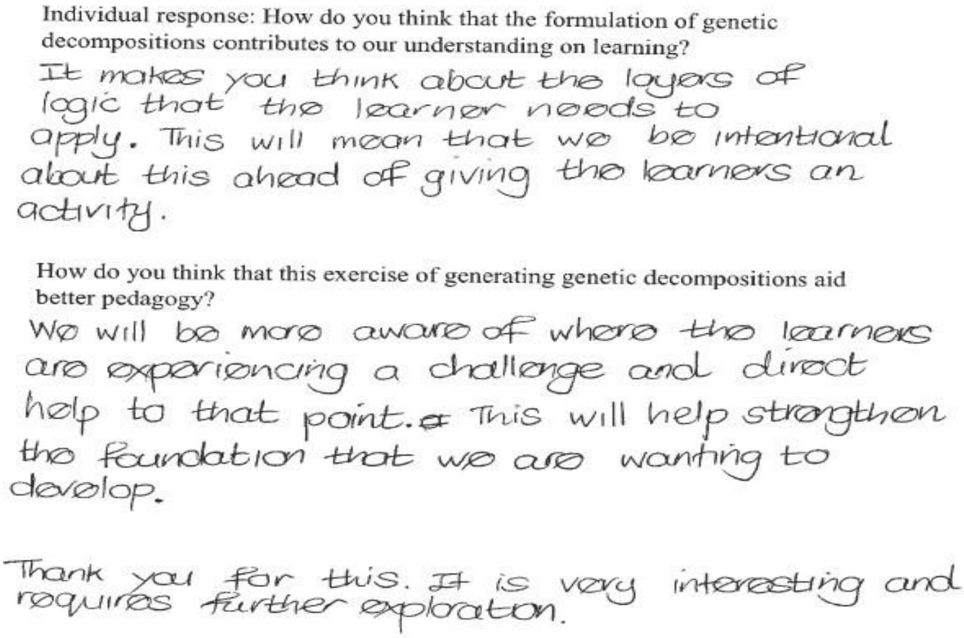
Data were collected from fourteen mathematics teachers and six physical sciences teachers, categorized according to Table 1. One mathematics teacher's response was unclear and therefore excluded, leaving data from thirteen mathematics teachers.

Twelve mathematics teachers reported that genetic decomposition contributes to learning, noting that it helps clarify what is required of students and enables them to be more intentional when designing tasks. Regarding teaching, ten mathematics teachers indicated that genetic decomposition is beneficial, with most emphasizing that it supports effective instruction and simplifies teaching. A sample response supporting genetic decomposition for both teaching and learning is provided in Scan 6. There was one neutral response on learning and three neutral responses on teaching. It is unclear whether these teachers misunderstood the question or were unable to express their views; a sample is also provided in Scan 7. Lastly, none of the teachers expressed views against the use of genetic decomposition in the teaching and learning of mathematics.

In Physical Sciences, all six teachers supported the use of genetic decomposition for learning. Overall, they indicated that it helps develop, break down, and simplify tasks for both teachers and learners. For teaching, only four teachers expressed support for genetic decomposition, noting that it helps them identify students' challenges and deliver lessons more effectively; a sample is provided in Scan 8. None of the teachers expressed negative views about using genetic decomposition in the teaching and learning of Mathematics. Lastly, there were no neutral responses regarding learning, and two neutral responses regarding teaching; a sample is provided in Scan 9.

Table 1: Teacher preference to use of genetic decompositions (F-favorable, U-unfavorable and N-neutral)

	Mathematics			Physical Sciences		
	F	U	N	F	U	N
Learning	12	0	1	6	0	0
Teaching	10	0	3	4	0	2



Individual response: How do you think that the formulation of genetic decompositions contributes to our understanding on learning?

It makes you think about the layers of logic that the learner needs to apply. This will mean that we be intentional about this ahead of giving the learners an activity.

How do you think that this exercise of generating genetic decompositions aid better pedagogy?

We will be more aware of where the learners are experiencing a challenge and direct help to that point. This will help strengthen the foundation that we are wanting to develop.

Thank you for this. It is very interesting and requires further exploration.

Scan 6: A response from Mathematics Teacher 2 (MT2) on the contribution of genetic decomposition to teaching and learning.

The response provided in Scan 6, by MT2 makes a clear justification for using GDs in the learning process. Firstly, the reference to the layers of logic co-inside with the hierarchical nature of APOS. This learning theory predicts that a process conception is attained after the interiorization of the action conception, and the object conception of a mathematical concept is attained through the encapsulation of a process conception. Secondly, MT2 hints that learning is a prerequisite for planning teaching. Learning activities need to be planned before learning occurs. Lastly, in the pedagogy responses, MT2 stated that GD helps teachers understand learners' challenges, thereby enabling them to identify where support is needed. In that regard, this means GD assists teachers in gaining a clearer understanding of the difficulty learners face during the process of knowledge construction.

Individual response: How do you think that the formulation of genetic decompositions contributes to our understanding on learning?

The question is clear to the learner

How do you think that this exercise of generating genetic decompositions aid better pedagogy?

An individual become aware of process as a totally and realises that transformation can act on it

Scan 7: A response from Mathematics Teacher 7 (MT7) on the contribution of genetic decomposition to teaching and learning.

The response provided in Scan 7 by MT7 is neutral regarding the contribution of GD to learning. However, with respect to pedagogy, MT7 stated that GD helps individuals become aware of a process as a complete entity that can be transformed. Shortly, MT7 indicated that GD is one of the approaches that supports individuals in reaching the mental stage referred to as Object.

Individual response: How do you think that the formulation of genetic decompositions contributes to our understanding on learning?

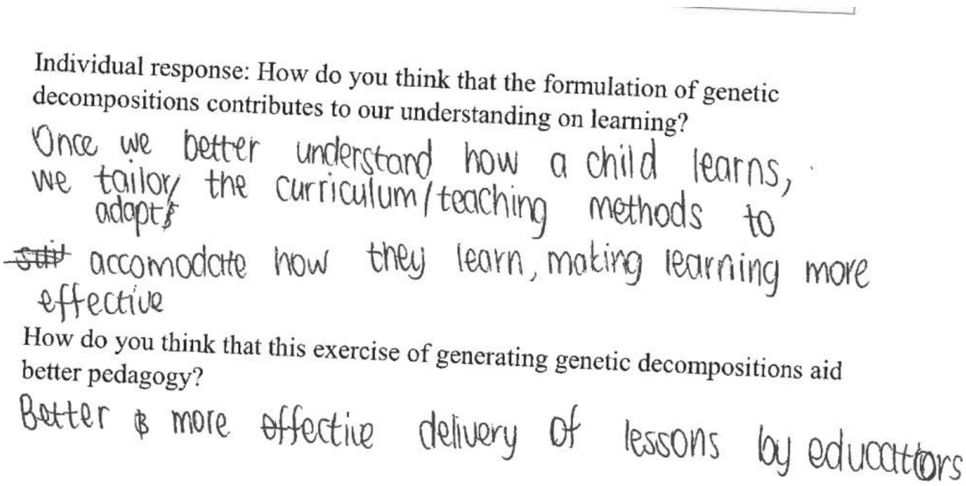
Assists in developing tasks for learners at different cognitive levels. Better understanding of learner misconceptions.

How do you think that this exercise of generating genetic decompositions aid better pedagogy?

Enables improved strategies to address learner difficulties to allow them to progress through higher levels of thinking.

Scan 8: A response from Physical Sciences Teacher 3 (PT3) on the contribution of genetic decomposition to teaching and learning.

In Scan 8, PT3 states that GD helps teachers design tasks that align with the mental constructions of different students during learning; hence, it enables them to better understand learners' misconceptions. Regarding pedagogy, PT3 states that adopting GD allows teachers to refine their teaching strategies so that learners' difficulties are addressed more efficiently. In this way, students can be more swiftly guided toward higher mental stages, such as the schema level.


Individual response: How do you think that the formulation of genetic decompositions contributes to our understanding on learning?

Once we better understand how a child learns, we tailor/adapt the curriculum/teaching methods to ~~suit~~ accomodate how they learn, making learning more effective

How do you think that this exercise of generating genetic decompositions aid better pedagogy?

Better & more ~~effective~~ effective delivery of lessons by educators

Scan 9: A response from Science Teacher 4 (PT4) on the contribution of genetic decomposition to teaching and learning.

In Scan 9, PT4 expresses a neutral stance regarding the contribution of genetic decomposition (GD) to the understanding of learning. In response, the teacher does not relate his argument to the mental constructions from APOS theory. The response is largely generic and therefore does not allow for clear conclusions to be drawn. With respect to pedagogy, PT4 states that GD assists teachers in delivering effective lessons. We believe this is so because GD provides a theory-driven framework for planning, sequencing, and implementing instruction that aligns with how students construct mathematical knowledge.

5. Conclusion

This study found that different APOS levels of conception are necessary at different stages of solving problems involving the tasks in Mathematics and Physical Science. We found that, from the group response of the Physical Science teachers that they could justify the action level of a GD but failed to provide sound justification for placing the process and object conceptions in accordance with APOS theory. The physical science teachers failed to identify the mechanisms by which a process or object can be attained. The findings arising from the analysis of the mathematics teachers' construction of a GD showed that the mental constructions from the PGD3 were similar. However, the justification for the specific APO mental construction could differ.

From the questionnaire, the data points out that there is a connection between logistical thinking during learning and the hierarchy of mental constructions in APOS. Also, data from teachers illustrated that learning activities using GDs should be planned before the actual lessons occur.

Teachers also indicated that GDs assist in teaching in that they 1) help teachers understand learners' challenges, thereby enabling them to identify where support is needed and 2) allows teachers to refine their teaching strategies so that learners' difficulties are addressed more efficiently. In this way, students can be more swiftly guided toward higher mental stages, such as the schema level. Teachers also indicated that for learning, GDs help: 1) in that learning is a prerequisite for planning teaching. Learning activities need to be planned before we know how learning occurs and 2) teachers design tasks that align with the mental constructions of different students during learning; hence, it enables them to better understand learners' misconceptions. This study provides a strong foundation for future research to explore the application of APOS framework across different mathematical topics and grade levels.

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