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A Six Sigma-Based Process Capability Analysis Model with Joint Confidence Blocks for Smaller-the-Better Characteristics

Chang-Hsien Hsu, Yin-Chieh Su*, Chen-Ning Chang

Abstract: Process capability indices (PCIs) are essential tools for quality assessment, yet existing methods face critical limitations. Traditional capability analysis relies on fixed classification standards (inadequate, capable, satisfactory, superior) that lack sufficient granularity for continuous improvement and do not align with Six Sigma methodology, now dominant in quality management. Furthermore, conventional approaches using point estimates fail to account for sampling uncertainty, potentially leading to erroneous capability conclusions. This study develops a Six Sigma-based smaller-the-better process capability analysis model (SBPCAM6σ) that addresses these limitations. The model establishes mathematical relationships between Six Sigma performance levels (3σ, 4σ, 5σ, 6σ) and integrated capability standards (C_o values) for multi-characteristic products, creating a framework with four parallel decision lines that enable graduated performance assessment. By integrating joint confidence block (JCB) methodology, the approach explicitly acknowledges sampling uncertainty rather than presenting spurious precision through point estimates. Application to a 92-octane gasoline quality control case demonstrates the framework's practical advantages. Compared with single-threshold methods, SBPCAM6σ provides more precise problem severity identification, transparent uncertainty communication, and graduated improvement targets aligned with organizational Six Sigma initiatives. The framework preserves valuable JCB methodology while extending capability analysis to support systematic continuous improvement with contemporary quality management practices.

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1. Introduction

Process capability indices (PCIs) have become essential quality assessment tools since Juran (1974) established their foundational framework. The smaller-the-better index C_{pu} is particularly relevant for quality characteristics where lower values indicate superior performance, such as defect rates, contamination levels, or dimensional deviations.

Modern products possess multiple quality characteristics that must simultaneously meet specifications. Chen et al. (2006) demonstrated that multi-characteristic products require individual characteristics to achieve higher capability levels than single-characteristic items. Ouyang et al. (2013) developed the integrated capability index C_0 to determine minimum individual characteristic requirements for multi-process products. Various multi-process capability analysis methods have emerged, including Singhal's (1990) MPPAC, Pearn and Shu's (2003) modified C_{pk} MPPAC, and Chen and Chen's (2007) AMPPAC. Hsu (2017) developed SBPCAM incorporating joint confidence blocks (JCB) to address sampling uncertainty in practical inspection scenarios.

Existing methods face a critical limitation: fixed classification standards categorizing processes as inadequate ($c < 1.00$), capable ($1.00 \leq c < 1.33$), satisfactory ($1.33 \leq c < 1.50$), or superior ($c \geq 2.00$) (Pearn and Chen, 1997). These standards present three drawbacks. First, four discrete levels lack granularity for continuous improvement. Second, large gaps between thresholds provide limited intermediate milestones. Third, fixed standards do not align with Six Sigma methodology's progressive levels (3σ, 4σ, 5σ, 6σ) (Harry & Schroeder, 2000; Pyzdek and Keller, 2014), now dominant in quality management. Despite Six Sigma's widespread adoption, no existing methodology systematically integrates these frameworks for smaller-the-better characteristics under sampling inspection.

This study develops SBPCAM6σ addressing these limitations through three objectives. First, we establish mathematical relationships between Six Sigma levels and integrated capability standards (C_0 values) for multi-characteristic products using C_{pu} = σ_level/3. Second, we construct a framework with multiple parallel decision lines enabling simultaneous performance assessment and improvement target identification across sigma levels. Third, we integrate JCB methodology (Hsu, 2017) ensuring reliable assessment under sampling inspection, acknowledging uncertainty rather than presenting false precision through point estimates (Chen et al., 2009).

2. Theoretical Framework

2.1 Review of Traditional SBPCAM

Hsu (2017) developed the smaller-the-better process capability analysis model (SBPCAM) to evaluate multi-characteristic products under sampling inspection. The model employs two indices: accuracy index $A = \mu/USL$ and precision index $P = \sigma/USL$, where μ is process mean, σ is standard deviation, and USL is upper specification limit. For smaller-the-better characteristics, both indices should be minimized.

Following Kane's (1986) original work on process capability indices, Hsu (2017) demonstrated that the process capability index C_{pu} can be expressed as:

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{1 - A}{3P}$$

This formulation captures both accuracy and precision in a single metric.

For multi-characteristic products with t characteristics, Ouyang et al. (2013) showed that achieving integrated capability c requires each characteristic to meet a stricter standard C_0:

$$C_0 = \frac{1}{3}\Phi^{-1}\left(\frac{[2\Phi(3C) - 1]^{1/t} + 1}{2}\right)$$

where Φ denotes the standard normal cumulative distribution function. As t increases, individual capability requirements increase to maintain overall conformance probability.

Setting C_{pu} = C_0 and substituting into the relationship above yields the decision line equation:

$$A + 3C_0 P = 1$$

Characteristics plotting left of this line meet the capability requirement; those plotting right require improvement.

Traditional SBPCAM uses four discrete standards (Chen et al., 2001; Pearn and Chen, 1997): inadequate (c=1.00), capable (c=1.33), satisfactory (c=1.50), and superior (c=2.00). A single decision line is constructed for the selected standard. This approach distinguishes adequate from inadequate performance but provides limited intermediate targets for continuous improvement.

2.2 Integration of Six Sigma Standards

Six Sigma methodology defines process capability in terms of sigma levels, which quantify how many standard deviations fit between the process mean and the specification limit (Harry and Schroeder, 2000; Pyzdek and Keller, 2014). For smaller-the-better characteristics with USL as the only specification limit, the relationship between sigma level and C_{pu} is straightforward:

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{\sigma_{\text{level}}}{3}$$

This yields: 3σ →C_{pu}=1.00, 4σ→C_{pu}=1.33, 5σ→C_{pu}=1.67, and 6σ→ C_{pu}=2.00. The 3σ, 4σ, and 6σ levels coincide with traditional standards while providing systematic graduated benchmarks aligned with Six Sigma methodology.

Applying these c values to the C_0 formula for multi-characteristic products yields the graduated standards presented in Table 1, which presents the minimum individual characteristic capability for each sigma level.

Table 1 The Process Capability Index C_o for Multi-Characteristic Products under Six Sigma Standards

t	3σ (c=1.00)	4σ (c=1.33)	5σ (c=1.67)	6σ (c=2.00)
1	1.000	1.330	1.670	2.000
2	1.068	1.384	1.714	2.037
3	1.107	1.414	1.739	2.059
4	1.133	1.436	1.757	2.074
5	1.153	1.452	1.770	2.085
6	1.170	1.465	1.781	2.095
7	1.183	1.477	1.791	2.103
8	1.195	1.486	1.799	2.110
9	1.205	1.495	1.806	2.116
10	1.214	1.502	1.812	2.121

Table 1 shows that C_0 increases with both the number of characteristics t and the target sigma level, enabling organizations to establish incremental improvement targets aligned with their quality objectives.

2.3 The SBPCAM6σ Framework

SBPCAM6σ employs four parallel decision lines. For a product with t characteristics:

3σ decision line: $A + 3C_{0(3\sigma)}P = 1$

4σ decision line: $A + 3C_{0(4\sigma)}P = 1$

5σ decision line: $A + 3C_{0(5\sigma)}P = 1$

6σ decision line: $A + 3C_{0(6\sigma)}P = 1$

where $C_0(k\sigma)$ denotes the C_0 value obtained from Table 1. These lines partition the A-P plane into regions corresponding to different performance levels. Figure 1 illustrates the framework for t = 5.

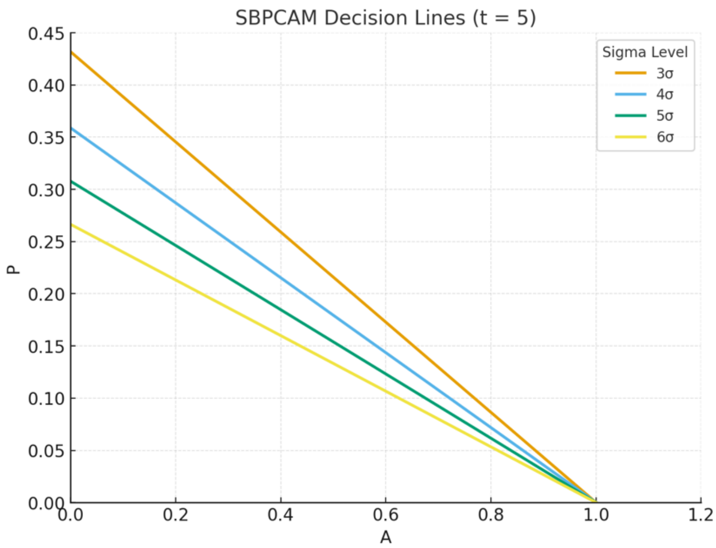

Figure 1 The SBPCAM6σ Framework for Products with Five Quality Characteristics (t = 5)

Characteristics plotting left of a line meet that sigma level. The framework provides graduated assessment, clear improvement targets, and natural integration with Six Sigma quality systems. Complete implementation procedures accounting for sampling inspection and estimation uncertainty are detailed in next section.

3. SBPCAM6σ under Sampling Inspection Plan

The SBPCAM6σ framework described above assumes known process parameters μ and σ, which are then used to calculate the accuracy index A = μ/USL and precision index P = σ/USL. However, in practical manufacturing environments, these parameters are rarely known with certainty and must be estimated from sample data collected through inspection procedures (Montgomery, 2009).

While 100% inspection provides complete information about the production lot and eliminates sampling error, it is often impractical or economically infeasible due to resource constraints, time limitations, and inspection costs. Consequently, sampling inspection plans are widely employed in quality control practice (Montgomery, 2009). Under sampling inspection, the observed sample mean x̄ and sample standard deviation s serve as point estimates for μ and σ, respectively. These estimates are subject to sampling variability, introducing uncertainty into the calculated capability indices.

Point estimates $\hat{A} = \bar{x}/USL$ and $\hat{P} = s/USL$ may lead to incorrect conclusions (Chen et al., 2009). A process truly meeting the required sigma level might be incorrectly classified as inadequate due to unfavorable sampling variation, or conversely, an inadequate process might appear acceptable due to a fortunate sample. This estimation uncertainty becomes critical when performance lies near decision lines, where small estimation errors can result in misclassification.

To address this, we employ joint confidence blocks (JCB) to incorporate estimation uncertainty. By constructing confidence regions for A and P simultaneously, we account for sampling variability and provide more reliable capability assessments than point estimates alone.

Consider a random sample of size n from a process with smaller-the-better characteristics. Under normality assumptions (Montgomery, 2009), simultaneous confidence intervals for A and P can be constructed.

According to Boole's inequality, the 100(1-α)% joint confidence block for indices A and P can be derived as (Hsu, 2017):

$$P\left\{\hat{A} - t_{\alpha_1/2}(n-1)\frac{\hat{P}}{\sqrt{n}} \le A \le \hat{A} + t_{\alpha_1/2}(n-1)\frac{\hat{P}}{\sqrt{n}}, \frac{(n-1)\widehat{P^2}}{\chi^2_{\alpha_2/2}(n-1)} \le P^2 \le \frac{(n-1)\widehat{P^2}}{\chi^2_{1-\alpha_2/2}(n-1)}\right\} \ge 1 - \alpha_1 - \alpha_2$$

where $t_{\alpha_1/2}(n-1)$ denotes the t distribution percentile, $\chi^2_{\alpha_2/2}(n-1)$ and $\chi^2_{1-\alpha_2/2}(n-1)$ denote χ^2 distribution percentiles, and $\hat{A} = \bar{x}/USL$, $\hat{P} = s/USL$.

Setting $\alpha_1 = \alpha_2 = \alpha/2$, the Cartesian product $S(X) = [A_1, A_2] \times [P_1, P_2]$ where:

$$A_1 = \hat{A} - t_{\alpha/4}(n-1)\frac{\hat{P}}{\sqrt{n}}, A_2 = \hat{A} + t_{\alpha/4}(n-1)\frac{\hat{P}}{\sqrt{n}}$$

$$P_1 = \left[\frac{(n-1)\hat{P}^2}{\chi^2_{1-\alpha/4}(n-1)}\right]^{1/2}, P_2 = \left[\frac{(n-1)\hat{P}^2}{\chi^2_{\alpha/4}(n-1)}\right]^{1/2}$$

The JCB represents a rectangular confidence region in the A-P plane containing the true values with probability at least 1-α (Hsu, 2017).

The implementation procedure integrates graduated sigma standards with JCB methodology (Hsu, 2017):

Step 1: Specification and Planning. Determine the total number of quality characteristics t and obtain C_0 values for all sigma levels (3σ, 4σ, 5σ, 6σ) from Table 1.

Step 2: Framework Construction. Construct the SBPCAM6σ chart by plotting four decision lines in the A-P coordinate plane using the equation $A + 3C_0(k\sigma)P = 1$ for k = 3, 4, 5, 6.

Step 3: Data Collection and JCB Calculation. For each quality characteristic, set the confidence level 1-α (typically $\alpha = 0.05$), collect a random sample of size n from the process, calculate sample mean $\bar{x}$ and sample standard deviation s, compute point estimates $\hat{A} = \bar{x}/USL$ and $\hat{P} = s/USL$, then calculate the JCB boundaries $[A_1, A_2] \times [P_1, P_2]$ using the formulas provided above.

Step 4: Graphical Representation. Plot each quality characteristic's JCB as a rectangular region on the SBPCAM6σ chart, where each rectangle represents the confidence region for that characteristic's true (A, P) location.

Step 5: Capability Assessment. Assess sigma level performance by examining the position of each JCB relative to the decision lines. If the entire JCB lies to the left of a decision line, the characteristic meets that sigma level with high confidence. If the entire JCB lies to the right of a decision line, the characteristic fails to meet that sigma level. If the JCB intersects a decision line, the assessment is inconclusive at the given confidence level, indicating the need for additional sampling.

Step 6: Improvement Prioritization. Prioritize improvement efforts by focusing on characteristics whose JCBs lie entirely to the right of the target sigma level or intersect critical decision lines, considering both the strategic importance of each characteristic and the effort required for enhancement.

This approach ensures that capability assessments account for sampling uncertainty (Chen et al., 2009) while providing clear guidance for continuous improvement aligned with Six Sigma principles (Pyzdek and Keller, 2014).

4. Numerical Example

To demonstrate SBPCAM6σ application, we employ the 92-octane gasoline quality control case from Hsu (2017). This fuel grade, commonly used in high-altitude regions, requires quality consistency for engine performance, fuel efficiency, and emissions control. Substandard quality causes engine knocking, vapor lock, deposit accumulation, and increased emissions.

The quality is characterized by five smaller-the-better attributes where lower values indicate superior performance:

1. 10% distillation temperature (x_1): Temperature at which 10% evaporates
2. 50% distillation temperature (x_2): Mid-point distillation temperature
3. 90% distillation temperature (x_3): Temperature at which 90% evaporates
4. End-point distillation temperature (x_4): Final evaporation temperature
5. Residue oil (x_5): Remaining non-volatile content

The manufacturer aims to assess whether current production capability meets 4σ performance standards (Pyzdek and Keller, 2014) while identifying improvement opportunities toward higher sigma levels.

A sampling inspection plan was implemented with sample size n = 20 for each characteristic at confidence level $1 - \alpha = 0.95$. The choice of n = 20 is justified by several considerations. First, from a statistical theory perspective, this sample size is adequate for reliable application of both the t-distribution (for estimating the mean) and χ^2-distribution (for estimating variance) in constructing JCBs, as both distributions achieve reasonable stability with $n \geq 20$ (Montgomery, 2009). Second, $n = 20$ represents a practical balance between statistical precision and resource constraints in industrial quality control settings. While larger samples would narrow confidence intervals, Montgomery (2009) demonstrates that samples of 20-25 observations provide sufficient precision for process capability studies while remaining economically feasible for routine quality monitoring. Third, this sample size aligns with established practice in the process capability literature; Hsu (2017) employed $n = 20$ in the original SBPCAM framework, and similar sampling plans are common in JCB applications (Chen et al., 2009). The resulting JCB width reflects both the inherent process variability and the sampling uncertainty at this sample size, enabling honest assessment of whether capability conclusions can be drawn with confidence or whether additional data collection is warranted.

With $t = 5$ characteristics, the C_0 values from Table 1 are: $C_0(3\sigma) = 1.153$, $C_0(4\sigma) = 1.452$, $C_0(5\sigma) = 1.770$, and $C_0(6\sigma) = 2.085$, yielding the corresponding decision lines $A+3.459P=1$, $A+4.356P=1$, $A+5.310P=1$, and $A+6.255P=1$ for the 3σ, 4σ, 5σ, and 6σ levels, respectively (Ouyang et al., 2013).

Table 2 presents upper specification limits and calculated JCB boundaries (Hsu, 2017) for accuracy and precision indices.

Table 1 Specifications and JCB Results for 92-Octane Gasoline Quality Characteristics

Characteristic	USL	A_1	A_2	P_1	P_2
10% distillation temp (x_1)	70	0.75	0.79	0.03	0.05
50% distillation temp (x_2)	121	0.74	0.77	0.03	0.05
90% distillation temp (x_3)	190	0.84	0.87	0.02	0.04
End-point distillation temp (x_4)	225	0.93	0.95	0.02	0.03
Residue oil (x_5)	2	0.48	0.55	0.06	0.11

Note: Temperature specifications in °C; residue oil in %.

Figure 2 displays the SBPCAM6σ chart with the four decision lines and the JCB rectangles for all five characteristics, revealing the uncertainty inherent in sampling-based evaluation.

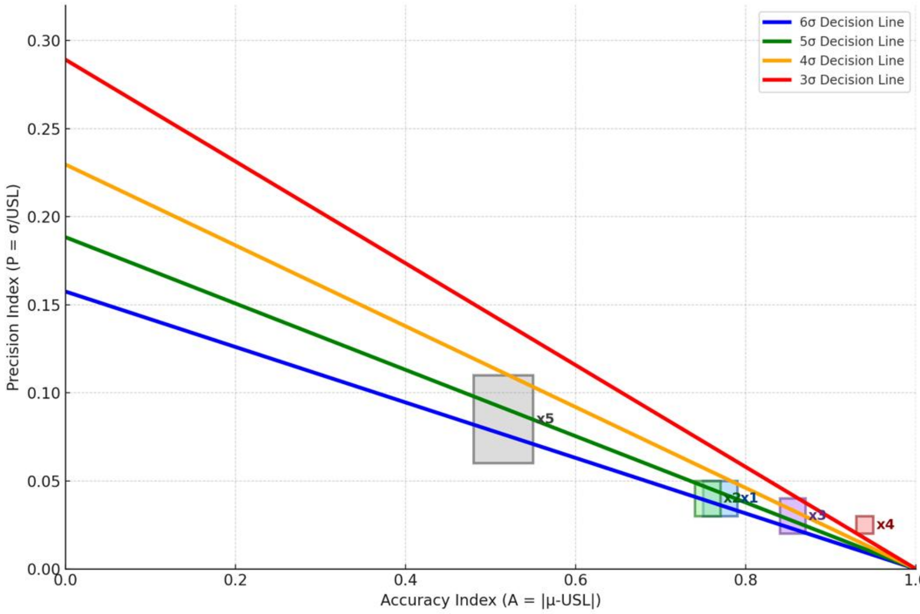

Figure 2 *SBPCAM6σ Analysis for 92-Octane Gasoline Quality Characteristics (t=5, n=20, σ=0.05)*

The JCB-based assessment yields three categories of results reflecting different confidence levels:

Characteristic x_4 (end-point distillation temperature): The JCB lies entirely right of the 4σ line, confirming failure to meet 4σ standards. More critically, the JCB intersects the 3σ line with three corners falling right of this baseline standard, indicating x_4 barely meets 3σ requirements. This represents the most severe capability deficiency, requiring urgent improvement.

Characteristic x_2 (50% distillation temperature): he JCB lies entirely left of the 4σ line, definitively meeting this level. However, the JCB intersects the 5σ line, leaving 5σ achievement uncertain.

Characteristics x_1, x_3, and x_5: These exhibit JCBs intersecting multiple decision lines. For x_1, the confidence block crosses 4σ, 5σ, and 6σ lines with corners in different performance regions. Similarly, x_3 and x_5 display JCBs spanning multiple sigma zones. With current sample size (n=20), precise sigma level performance cannot be definitively determined.

JCB-decision line intersections might initially appear limiting. However, this reflects the method's fundamental strength: honest reporting of statistical uncertainty. Point estimates using $\hat{A} = \bar{x}/USL$ and $\hat{P} = s/USL$ would suggest x_1, x_2, and x_5 all pass 5σ standards while x_3 passes 4σ but fails 5σ—conclusions that appear definitive but ignore sampling variability (Hsu, 2017). The JCB analysis reveals such certainty is unwarranted, with most characteristics requiring additional data before confident sigma level assignment.

This conservative approach prevents two critical errors: (1) falsely concluding adequate performance when capability remains uncertain, and (2) incorrectly identifying deficiencies based on sampling artifacts rather than genuine process issues.

The SBPCAM6σ with JCB analysis provides a structured framework for prioritizing quality improvement actions while appropriately accounting for statistical uncertainty (Pyzdek and Keller, 2014):

Priority 1: Urgent Improvement Required (x_4). End-point distillation temperature requires immediate attention. Analysis confirms failure to meet 4σ and barely achieves 3σ performance. This critical quality issue requires comprehensive process investigation. Priority areas include: column temperature control system evaluation, heating rate stabilization through improved energy management, raw material composition variability reduction, and operator procedure standardization. The improvement trajectory: first ensure consistent 3σ compliance, then advance toward 4σ, ultimately targeting 5σ. Given current sub-3σ borderline performance, initial efforts should focus on fundamental process stabilization.

Priority 2: Require Additional Data Collection (x_1, x_3, x_5). These characteristics display JCB-decision line intersections, indicating insufficient evidence for definitive sigma level determination. Two approaches: increase sample size beyond n = 20 to narrow confidence regions, or implement continuous monitoring to accumulate data over time. Until definitive data are available, manage these conservatively—assuming higher sigma levels (5σ or 6σ) are not achieved until proven otherwise.

Maintain and Document (x_2). The 50% distillation temperature definitively meets 4σ standards. Document and maintain current control practices. If targeting 5σ performance, additional sampling would determine achievement of this higher standard.

Methodological Implications. This case demonstrates three critical advantages: First, honest uncertainty quantification (Chen et al., 2009) - explicitly identifying which conclusions are statistically supported and which require additional evidence. This transparency supports better resource allocation. Second, graduated sigma standards offer clear improvement trajectories with concrete objectives rather than vague directives. Third, the approach prevents decision errors from sampling variability, guarding against both false positive and false negative conclusions.

"Inconclusive" is not method failure but appropriate acknowledgment of statistical reality. In quality management, making no decision until sufficient evidence is available is preferable to making confident but incorrect decisions based on inadequate data.

Applying SBPCAM6σ to the same dataset enables direct comparison with Hsu's (2017) analysis. Hsu employed a single decision line (c = 1.33, C_0 = 1.452), concluding only x_4 required improvement while x_1, x_2, x_3, and x_5 met capable standard. Our graduated approach reveals important differences summarized in Table 3.

Table 3 Comparison of Hsu (2017) and SBPCAM6σ Approaches

Aspect	**Hsu (2017)**	**SBPCAM6σ (This Study)**
Decision Lines	Single (c=1.33)	Four (3σ, 4σ, 5σ, 6σ)
Performance Standards	Fixed "capable"	Graduated Six Sigma
x_4 Assessment	Fails capable	Barely meets 3σ (critical)
x_2 Assessment	Meets capable	Confirmed 4σ
x_1, x_3, x_5 Assessment	Meet capable	Inconclusive (n=20 insufficient)
Improvement Guidance	Binary deficiency identification	Graduated targets with uncertainty acknowledgment

SBPCAM6σ provides three key enhancements: (1) graduated assessment revealing x_4's deficiency is more severe than single-threshold evaluation suggests—barely meeting 3σ rather than simply failing 4σ; (2) transparent uncertainty through explicit JCB-decision line intersections, indicating that while Hsu concluded x_1, x_3, and x_5 definitively met standards, current sample size yields inconclusive assessments; and (3) aligned improvement planning through Six Sigma targets integrating naturally with organizational quality management systems. These enhancements preserve Hsu's valuable JCB methodology while extending capability analysis to support systematic continuous improvement aligned with contemporary quality frameworks.

5. Conclusions

This study developed SBPCAM6σ, integrating Six Sigma graduated standards with joint confidence block methodology for multi-characteristic process capability assessment. The framework replaces traditional fixed classifications with four parallel decision lines (3σ, 4σ, 5σ, 6σ), enabling systematic performance evaluation aligned with contemporary quality management practices.

The SBPCAM6σ extends Hsu's (2017) original framework by incorporating graduated benchmarks rather than single-threshold evaluation. Through the relationship C_{pu} = σ_level/3, we derived C_0 values for each sigma level across t = 1 to 10 characteristics. JCB integration ensures assessments acknowledge sampling uncertainty rather than presenting false precision. The 92-octane gasoline case study demonstrated three practical advantages: revealing problem severity more precisely (x_4 barely meets 3σ, not merely failing 4σ), explicitly identifying inconclusive assessments requiring additional data, and providing graduated improvement targets aligned with Six Sigma methodology.

The research contributes theoretically by bridging capability indices and Six Sigma frameworks, moving process evaluation from binary classification toward continuous performance measurement. Methodologically, explicit JCB-decision line intersections transform uncertainty from implicit limitation to explicit decision criterion. Practically, the framework provides actionable guidance—clear statistical targets, visual assessment of multiple characteristics, and natural integration with organizational quality systems.

Three limitations warrant acknowledgment. First, normality assumptions may not hold for all quality characteristics. Second, higher sigma levels do not universally represent optimal targets; cost-quality considerations may justify 4σ as appropriate endpoints in specific contexts. Third, typical sample sizes (n = 20) frequently yield JCB-decision line intersections, producing inconclusive assessments that require either larger samples or acceptance of uncertainty.

Future research should pursue two directions. First, developing optimal sample size determination methods that balance statistical confidence with resource constraints would enhance practical applicability, particularly for characteristics exhibiting JCB-decision line intersections. Second, extending the framework to nominal-the-better and larger-the-better characteristics would provide comprehensive capability analysis across all quality attribute types (Spiring et al., 2003), creating a unified Six Sigma-based assessment methodology.

The SBPCAM6σ advances process capability analysis by aligning traditional assessment with Six Sigma practices while maintaining statistical rigor. The framework provides quality practitioners with enhanced decision-making tools that acknowledge uncertainty explicitly, support graduated improvement planning, and integrate naturally with contemporary quality management systems.

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Chang-Hsien Hsu, Yin-Chieh Su*, Chen-Ning Chang

Department of Business Administration, Asia University, Taichung City 41354, Taiwan

(*Corresponding author:111231002@live.asia.edu.tw)