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Can Machine Learning Assist Anomaly-oriented Portfolio Optimization Strategies?

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Abstract. This paper investigates the effectiveness of machine learning (ML) in enhancing portfolio optimization strategies for Shariah-compliant (SC) stocks in the United States. Utilizing LASSO regressions, we propose the identification of asset pricing anomalies among SC stocks and construct four distinct portfolio strategies: a conventional market portfolio, a SC market portfolio, an anomaly-based value-weighted portfolio, and an anomaly-based portfolio optimized using STATA's ovport command. Empirical results from 2000 to 2023 reveal that anomaly-oriented strategies, particularly those using ML variance optimization techniques, consistently outperform traditional benchmarks in terms of Sharpe ratio and cumulative returns. Our findings further suggest a positive relationship between the intensity of pricing anomalies—measured by the t-statistics of alpha—and portfolio performance. These results highlight the potential for ML-assisted anomaly detection to deliver superior, risk-adjusted returns while adhering to ethical investment principles. Policy implications include the need for Islamic finance regulators and fund managers to consider integrating ML-based anomaly screening into portfolio construction. While promising, the study acknowledges limitations including reliance on in-sample results and potential overfitting. Future research should emphasize out-of-sample validation, explore alternative ML models, and assess real-world constraints such as liquidity and transaction costs. Overall, this paper contributes to both the Islamic finance literature and the growing intersection of machine learning and ethical investing.

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1. Introduction and background

One of the foundational principles in finance is Modern Portfolio Theory (MPT), which suggests that investors should construct efficient portfolios that offer the optimal trade-off between risk and return, tailored to their individual risk appetite. Much of the discussion of MPT involves the portfolio selection problem. The portfolio selection problem was first mathematically formulated in the pioneering work of Markowitz (1952), which transformed the concept of investment diversification into a computational framework. Over the past 40 years, significant advancements have been made beyond the traditional Mean–Variance (MV) model introduced by Markowitz. Despite being a cornerstone of finance theory, the MV model has faced criticism for its high computational complexity and the challenge of estimating its parameters (including expected returns, variances, and covariances), limiting its practical application. This limitation has led to numerous efforts in the literature to linearize the quadratic objective function (Stone, 1973).

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Today, MV models comprising thousands of assets can be efficiently solved, significantly enhancing the practical utility of the MV approach for constructing large-scale portfolios. Real-time solutions are now feasible through the implementation of interior point algorithms for quadratic programming problems (Takehara, 1991) or by employing compact factorizations and piecewise linear approximations (Markowitz et al., 1993).

In this paper, we demonstrate that the portfolio selection and formation process can be simplified and made statistically significantly more profitable by constraining the universe of investable assets to groups of Shariah compliant (SC) stocks exhibiting anomalous asset pricing behavior. In order to simplify our analysis, we limit our dataset into US stocks only, but our analysis is easily replicable to other stock markets as well.

Asset pricing anomalies occur when the realized returns of a stock or portfolio consistently diverge from the expected returns predicted by its associated risk premiums. This mismatch enables investors to achieve excess returns beyond what is justified by risk, commonly known as "alpha," which is often identified through significant intercepts in asset pricing models (Chou et al., 2006). Expected returns are typically estimated using asset pricing models such as the Capital Asset Pricing Model (CAPM), Fama and French's three-factor model (Fama & French, 1992), five-factor model (Fama & French, 2015), and six-factor model (Fama & French, 2020). Over the years, substantial research has explored various asset pricing anomalies, as noted by Fama and French (2008, 2016).

On the other hand, SC stocks represent publicly-listed companies that adhere to Islamic principles by avoiding involvement in activities prohibited under Sharia law, such as alcohol production, gambling, interest-based financial services, and pork-related businesses (El-Gamal, 2006). In the US, identifying SC stocks involves rigorous qualitative and quantitative screening, including financial ratio criteria that restrict interest-bearing debt levels and cash holdings (Khatkhatay & Nisar, 2007; Sukor & Abdul Halim, 2022b).

Prominent indices, such as the MSCI Islamic Index Series and the S&P Dow Jones Islamic Market (S&PDJIM) Index, publish biannual lists of SC stocks to guide investors (Derigs & Marzban, 2008). As ethical and socially responsible investing (SRI) continues to gain momentum, SC stocks increasingly align with the principles of sustainable and ethical finance. They also resonate with the United Nations' Sustainable Development Goals (UN SDGs), particularly in promoting responsible investment practices (Siddique et al., 2022).

Since standard (quadratic or linear) portfolio optimization models typically select fewer than 20 securities, the procedures proposed in this paper are designed around the concept of constructing and solving mixed integer subproblems that focus on subsets of available investment choices. These subsets are generated by leveraging information from the relaxed problem, specifically the selected securities and reduced costs. A key advantage of these procedures are their generality, allowing for adaptation to various mixed integer models that may be relevant in portfolio selection, such as those incorporating transaction costs. The proposed algorithms can be directly applied to such models or easily generalized for similar applications.

We propose using LASSO regressions in identifying anomalies and subsequently compare and contrast the performance of four distinct portfolio optimization strategies: the simple market portfolio strategy, the simple SC market portfolio strategy, the value-weighted anomaly-oriented portfolio strategy and finally, the same anomaly-oriented strategy but using STATA's ovport command to decide the portfolio weights. Our findings indicate that the anomaly-oriented portfolio as well as anomaly-oriented portfolio using ovport consistently outperforms the other strategies.

Moreover, we observe a positive relationship between the t-statistics of the anomaly and portfolio performance, where higher t-statistics correspond to superior performance, and vice versa. Even the most basic anomaly-based strategies outperform the simple market diversification strategy.

The remainder of this paper is structured as follows; Section 1 provides the introduction and background, Section 2 discusses the prior literature relevant to this field, Section 3 elaborates on the problem statement and research framework, Section 4 explains the procedures employed in detail, Section 5 discusses the results from the procedures employed, Section 6 briefly demonstrate out-of-sample results for robustness and finally Section 7 provides the conclusory remarks.

2. Literature Review

The application of ML and AI in finance is generally divided into three categories; portfolio optimization, stock price movements and financial sentiment analysis. We provide a brief of each category in this section, followed by some discussions of SC stocks and asset pricing anomalies towards the end.

Beginning with portfolio optimization, which aims to construct portfolios that maximize returns while minimizing risk. Traditional methods like the Mean-Variance model proposed by Markowitz focus on variance as a risk measure. However, this approach often faces criticism for treating both upside and downside deviations equally. To address these limitations, modern AI-based portfolio optimization models integrate more sophisticated risk measures, such as Conditional Value at Risk (CVaR) and Semi-Absolute Deviation (SAD). Additionally, various optimization methods, including exact, heuristic, and hybrid approaches, have been developed to address the complexities involved. For instance, the Mean-Absolute Deviation (MAD) model minimizes the absolute deviation from the mean return while incorporating constraints such as cardinality and transaction lots (Mansini & Speranza, 1999; Speranza, 1996). AI techniques, such as Particle Swarm Optimization (PSO), Genetic Algorithms (GA), and Swarm Intelligence algorithms, are frequently employed to solve these complex models efficiently. Multi-objective models, which optimize for multiple goals like maximizing return and minimizing risk, have also gained traction. Recent advances include hybrid models that combine evolutionary algorithms with heuristic methods, offering practical solutions for large-scale portfolio management. By leveraging AI, portfolio optimization becomes more adaptive and capable of handling the dynamic nature of financial markets, ultimately leading to more robust investment strategies.

Then, stock market prediction is a challenging task due to the inherent volatility and complexity of financial markets. Traditional statistical methods, such as time-series analysis, are often insufficient for capturing non-linear and dynamic market behaviors. As a result, AI techniques have gained prominence, especially methods like Recurrent Neural Networks (RNN) and Long Short-Term Memory (LSTM) networks, which excel in processing sequential data (Nabipour et al., 2020). These models are particularly effective when integrating both historical market data and sentiment data from news or social media. For instance, using RNNs to process time-series data along with textual data from financial news can significantly improve prediction accuracy. Some studies combine fundamental indicators with machine learning models, such as using Support Vector Machines (SVM) to classify stock price movements based on log-returns and technical indicators. Additionally, hybrid models that combine genetic programming with neural networks have been shown to outperform traditional forecasting methods. Despite these advancements, challenges persist, such as the risk of overfitting and the difficulty of interpreting complex AI-driven predictions. Nevertheless, integrating AI into stock market prediction allows for more accurate forecasting, which is crucial for algorithmic trading and portfolio management.

Finally, Sentiment analysis in finance involves extracting insights from textual data, such as news articles, social media posts, or financial reports, to predict market movements. The rise of social media platforms like Twitter has made sentiment data abundant, prompting the use of Natural Language Processing (NLP) techniques to assess public sentiment regarding stocks. One common approach involves combining sentiment scores with volatility prediction models to enhance accuracy. For instance, the SAVING model utilizes Recurrent Neural Networks (RNN) to evaluate sentiment intensity and integrates these insights into volatility forecasts. Research has shown that models incorporating sentiment data, like RNN-LSTM, often outperform traditional methods that rely solely on historical price data. Furthermore, sentiment analysis is increasingly used in combination with network analysis to capture collective investor sentiment. For example, analyzing tweets from financial communities, such as StockTwits, has revealed that sentiment indicators can serve as reliable predictors of short-term market trends. Integrating sentiment analysis with financial models offers a more comprehensive understanding of market dynamics, as it not only captures quantitative data but also reflects investor psychology, leading to better-informed investment decisions (Batra & Daudpota, 2018).

Next, we provide a brief discussion on asset pricing anomalies. Asset pricing anomalies occur when the realized returns of a stock or portfolio consistently differ from the expected returns predicted by the associated risk premiums. This inconsistency enables investors to generate excess returns beyond those warranted by risk, commonly known as "alpha." In asset pricing models, alpha is typically represented by significant intercepts (Chou et al., 2006).

Expected returns are usually estimated using various asset pricing models, including the Capital Asset Pricing Model (CAPM), the Fama and French three-factor model (Fama & French, 1993), the five-factor model (Fama & French, 2015), and the six-factor model (Fama & French, 2020). Over time, numerous studies have investigated different asset pricing anomalies, with significant contributions made by Fama and French (2008, 2016).

Then, asset pricing anomalies studies for SC stocks in the US is thus far shown to be scarce. Abdul Halim and Sukor (2025) and Zaremba et al. (2020) are amongst those that focus on anomalous asset pricing behavior of US SC stocks. Since anomalous asset pricing behaviour entails superior returns given its risks, some justification should be given as to why US SC stocks should show such anomalous behaviour. We provide these reasons here;

In theory, a genuinely Sharia-compliant (SC) firm should be founded on the principle of profit sharing, which fundamentally involves a predetermined profit-sharing ratio between the capital owners and the firm's management (Anwer et al., 2021). During favorable or unfavorable economic conditions, US SC stocks may strive their best to sustain stable profit levels and adhere to the predetermined profit-sharing ratio. This practice can create a pathway for statistically significant alphas, akin to the Profit Equalization Reserve used by contemporary Islamic banks (AAOIFI, 2018; Sukor & Abdul Halim, 2022b).

At the same time, Sukor and Abdul Halim (2022b) indicate that SC firms in the US—or in countries lacking robust Islamic capital markets—often display distinct financial behaviors, such as limited leverage, larger size, higher profitability, lower investment, and low book-to-market ratios. More importantly, these skewed corporate financial characteristics may lead to an above-average internal rate of returns. These characteristics, which align with contemporary asset pricing models, inherently influence the asset pricing behavior of US SC firms. Additionally, US SC stocks, by definition, are constrained in terms of accounts receivables and cash holdings, which can skew the behavior of current operating assets. This skewed pattern, when combined with an above-average internal rate of return, may result in potential asset pricing anomalies (Avramov et al., 2019; De Groot & Huij, 2018; Zaremba, 2016).

Investor preferences also play a significant role in shaping the asset pricing behavior of SC stocks. Besides conventional investors, US SC stocks attract at least two additional groups: Islamic funds (such as OIC sovereign wealth funds) and Sharia-conscious retail investors. These investors, guided by religious mandates or ethical considerations, restrict their investments to SC stocks exclusively. The collective impact of these groups, combined with conventional investor demand, may lead to unique pricing characteristics, influencing the cost of capital and corporate financial practices, particularly regarding accruals.

Furthermore, increased investor interest can enhance liquidity and analyst coverage, improving informational efficiency and further differentiating SC stocks from their conventional counterparts. Previous studies have shown that higher informational efficiency in SC stocks can influence stock price reactions to the "Islamic label", as seen in markets like Malaysia (Bacha & Abdullah, 2001), Egypt (Sadeghi, 2008), and Jordan (Hayat & de Anca, 2017).

Moreover, the SC investment universe deliberately excludes entire sectors deemed non-compliant, particularly those associated with "sinful" activities such as gambling, alcohol consumption, and, notably, the financial industry. This exclusion can influence broader asset pricing behavior of SC stocks, potentially affecting the occurrence of significant alphas (Abdul Halim, 2023; Moskowitz & Grinblatt, 1999; Zaremba & Umutlu, 2018). While this limited investment base may reduce diversification, the inclusion of a broader range of ethically conscious investors could lead to a distinct demand distribution compared to conventional US stocks.

Given these factors, it is reasonable to expect US SC stocks to exhibit higher informational efficiency and lower information asymmetry, which could reduce the likelihood of significant alphas. However, as highlighted by Fama and French (2015), studies analyzing monthly excess returns typically assume that relevant information is already incorporated into US SC stock prices. Therefore, any observed significant alphas are likely attributable to abnormal risk-adjusted returns, indicating potential market inefficiencies.

Finally, research by Haseeb et al. (2022) and Farooq and Ahmed (2024) indicates that SC stocks in emerging economies tend to experience a lower risk of price crashes and higher stock price synchronicity. These findings are linked to the distinct financial traits of SC stocks and the increased scrutiny resulting from a broader investor base. Extrapolating from these results, it is reasonable to hypothesize that US SC stocks may exhibit similar patterns regarding accruals and asset pricing behavior.

3. Problem statement and research framework

Naturally, we now ask the question, can asset pricing anomalies amongst US SC stocks be a guidance for ML-based portfolio optimization? More specifically, can an anomaly-oriented portfolio outperform traditional investment portfolios?

These questions arise from several gaps identified in the current literature. First, there is a noticeable scarcity of studies focused on portfolio optimization using US Sharia-compliant (SC) stocks. While the field of portfolio optimization is well-established, most existing research predominantly targets conventional stocks, leaving SC stocks relatively underexplored. Given the growing interest in Islamic finance and the increasing number of SC assets in the US market, addressing this gap is both timely and relevant.

Second, the application of Artificial Intelligence (AI) and Machine Learning (ML) techniques to US SC stocks is even more limited. Despite the proven potential of AI/ML in enhancing portfolio management, few studies have specifically focused on SC stocks within this context. Integrating AI/ML approaches with SC portfolio optimization can offer unique insights, especially when addressing the distinct financial and ethical constraints characteristic of SC investing.

Lastly, employing anomaly-oriented techniques represents a novel approach that not only advances the field of SC equity investments but also holds significant implications for general stock investments. By leveraging anomalies—such as the dividend anomaly—as signals within the optimization framework, this approach contributes to more robust portfolio construction methods. Such an innovative methodology is valuable not only within Islamic finance but also for broader applications in asset pricing and investment strategies.

Our research question to answer this problem statement is therefore:

Can ML techniques assist anomaly-oriented strategies for US SC stocks?

To help answer our research question, we propose the following research objectives:

To investigate the difference in dynamic portfolio performance of anomaly-oriented ML strategy versus traditional strategies.

To meet this research objective, we propose the following research hypothesis:

Using anomaly-oriented strategies together with ML assistance leads to superior portfolio Sharpe ratio.

4. Data and methodology

4.1 Automation Using LASSO Regressions

To automate the discovery of anomalous portfolios, we suggest the use of LASSO regression techniques. LASSO (Least Absolute Shrinkage and Selection Operator) is a penalized regression that performs variable selection by shrinking some coefficients to exactly zero. In our context, we propose the regression of the stock's monthly average excess returns on a large set of candidate risk factors.

We provide here a typical STATA command to fit a LASSO model, which may be tailored to many specifications as need be:

```
lasso linear ret mktrf smb hml rmw cma mom... Fp, selection(cv)
```

where ret is the average excess returns for sorted portfolios of US SC stock's return and mktrf smb hml rmw cma mom... Fp are risk factor variables.

The potential researcher is free to choose the set of risk factor variables to include the LASSO regressions, however, we suggest a broad set of factor variables instead of a narrow set, since LASSO will automatically shrink the coefficients of unnecessary independent variables (zero) to zero if they are not important to the explanation of the dependent variable (average excess returns). Using a narrow set of risk factor variables however, may run the risk of model misspecification (as usually the case for the CAPM, see Abdul Halim & Sukor (2025)) which may result in an invalid model.

```
lasso linear y x1 x2 ... xn, selection(cv)
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Cross-validation (selection(cv)) is commonly used to choose the penalty, which in this case, a 10-fold cross-validation was used to determine the optimal penalty parameter, as is the standard setting in STATA. After fitting, one can use lasso predict to generate out-of-sample predictions or to identify which factors remain with nonzero coefficients.

The advantage of LASSO regressions are its built-in feature selection: it will pick out a sparse set of risk factors with the strongest explanatory power, automatically discarding weak or redundant ones. This helps prevent overfitting and handles cases where the number of candidate anomalies is large. (By contrast, traditional stepwise regression or brute-force searches can be cumbersome when there are many factors.) Another benefit is regularization: LASSO regression shrinks coefficients toward zero, which can improve out-of-sample robustness.

However, LASSO regressions have its limitations. When predictors are highly correlated, LASSO regressions may arbitrarily select one and ignore others, potentially missing relevant variables. Its selection is also sensitive to the choice of penalty parameter and requires careful validation. Moreover, like any regression, LASSO regressions assume linear relationships and may omit nonlinear effects or interactions. In practice, LASSO regressions are primarily used as a screening tool: once the most predictive anomalies are identified, we can then build more elaborate models or portfolios based on that subset. More importantly, the statistical significance of alphas should be recorded. In this paper, a highly significant alpha suggests anomalous portfolios, a non-significant alpha suggest non-anomalous portfolios, and a weakly significant alpha (with t-statistics between 1.8 to 1.6) is considered as a weakly anomalous portfolio.

We utilize the findings of Abdul Halim & Sukor (2025) as our source of anomalous portfolios that show signals of anomalous, non-anomalous and weakly anomalous asset pricing behavior. Next using these portfolios, we present our for main procedures for portfolio construction below.

4.2 Portfolio Construction Methodology

Procedure 1: Conventional Diversified Portfolio

We begin by constructing a conventional diversified portfolio comprising all publicly listed stocks on NYSE, AMEX, and NASDAQ. Each stock i in the universe U is assigned a weight proportional to its market capitalization following the standard value-weighted portfolio construction methodology:

$$w_i = \frac{MC_i}{\sum_{j=1}^{\mathrm{U}} MC_j}$$

here MC_i is firm i's market cap. The portfolio is rebalanced annually to maintain these target weights (and to incorporate changes such as new listings or delistings). Annual rebalancing ensures that weights reflect current market values while keeping transaction costs minimum. For this paper, we employ the S&P 500 Index as the proxy for our conventional diversified portfolio.

The rationale for value-weighting is that market cap reflects each firm's size and economic importance, and it yields a straightforward "market portfolio" benchmark. However, this approach has well-known limitations amongst them are value-weighted portfolios tend to be skewed towards the stock price movements of the largest firms, since these carry the highest market capitalization Mansini (1999). In practice, a few mega-cap stocks will often dominate the portfolio's return, and the influence of smaller firms is relatively muted. Moreover, a pure cap-weighted portfolio is insensitive to many known return anomalies: if high expected returns reside in small- or mid-cap stocks (or other low-cap segments), the value-weighted portfolio will underweight them and thus may miss valuable alpha. In short, while easy to implement, the conventional market capitalization value-weighted strategy can suffer from concentration risk and may be less effective at harvesting alpha signals than alternative weighting schemes.

Procedure 2: Shariah-Compliant Diversified Portfolio

Procedure 2 repeats the above steps but restricts the investment universe to the universe of SC stocks only. That is, we first screen the full universe U for Shariah compliancy using standard S&P Dow Jones (2019) methodology. Shariah screening involves two tiers: the first tier involves a qualitative filter excluding firms involved in prohibited industries (such as alcohol, gambling, pornography, pork products, conventional banking/financial services, and other prohibited business areas), the second tier on the other hand involves quantitative filters which typically impose a set of maximum thresholds on certain financial ratios. For example, common quantitative criteria require that a firm's interest-bearing debt or accounts receivable do not exceed specified fractions of market value (often on the order of 30–33%), and that non-operating (interest) income is below a modest threshold (e.g. 5%) of market value. In practice, we represent the screening rule as: if stock i meets all qualitative requirements and all quantitative ratios fall below their respective thresholds (collectively known as θ), then it is deemed SC. Formally, we let

$$S_i = \begin{cases} 1, if\ stock\ i\ passes\ qualitative\ screen\ and\ \theta \\ 0, \qquad otherwise \end{cases}$$

where S_i=1 indicates a SC stock.

We apply these screens to all stocks in U to obtain the SC subset U_{SC}. Once U_{SC} is defined, we construct a SC diversified portfolio exactly as in Procedure 1, but using only the SC stocks. That is, within U_{SC} we assign each firm *i* a weight $w_i = \frac{MC_i}{\sum_{j=1}^{\mathrm{U}_{SC}} MC_j}$, and rebalance the portfolio yearly. The purpose of this procedure is to isolate the performance of a broad Islamic-equity portfolio, using the same market-cap-weighting and rebalancing conventions, but adhering to two-tier Shariah stock screening procedure. For this paper, we employ the S&PDJ Islamic Index US as the proxy for our SC diversified portfolio.

Procedure 3: Anomaly-Based Value-Weighted SC Portfolios

In Procedure 3, we leverage asset pricing anomalies to create targeted portfolios from the SC universe. First, for each Shariah-compliant stock $i \in \mathrm{U}_{SC}$ we estimate its abnormal return (alpha, if any) and associated *t*-statistic, t_i. This is automated by the previously discussed LASSO regressions procedure. We then form three groups of stocks based on the value of t-statistics of alpha, t_i: those with strongly significant alpha, marginally significant alpha, and insignificant alpha. Concretely, let T_h and T_m be high and medium significance thresholds (e.g. T_h≥2, T_m≈1.6 for two-tailed tests at 5% significance level). We define indicator variables for group membership as follows:

$$G_{i1} = 1(|t_i| > T_h)$$
$$G_{i2} = 1(T_m < |t_i| < T_h)$$
$$G_{i3} = 1(|t_i| < T_m)$$

In words, Group 1 consists of stocks with the highest *t*-statistics (strong anomaly signals), Group 2 has intermediate *t*-statistics, and Group 3 contains stocks whose estimated alpha is not statistically different from zero.

After classifying the SC stocks into these three anomaly groups, we construct a separate value-weighted portfolio for each group. Within Group *K*, each stock *i* is assigned weight $w_i = \frac{MC_i}{\sum_{j=1}^{G_k} MC_j}$. These value-weighted anomaly portfolios are rebalanced annually like before. The economic rationale is that higher *t*-statistic (Group 1) stocks appear to be more mispriced relative to the benchmark model and thus represent strongly anomalous portfolios, so one expects them to offer larger exploitable alphas. In contrast, Group 3 stocks serve as a non-anomalous benchmark. By sorting on *t*-stats, we effectively create portfolios that target increasing levels of anomaly intensity.

Procedure 4: Anomaly Portfolios Optimized Using *ovport*

Procedure 4 replaces the value-weighting in Procedure 3 with mean-variance optimization. Specifically, instead of setting weights proportional to market cap within each anomaly group, we use Stata's ovport command to compute an optimal variance (tangency) portfolio from each group (Sukor & Abdul Halim, 2022a). Under modern portfolio theory (MPT), we solve for weights *w* that optimize the trade-off between expected return and variance. For example, ovport finds the portfolio that maximizes the Sharpe ratio given a risk-free rate R_f. Formally, the optimization is:

$$Max_w \frac{w\prime(\mu-R_f 1)}{\sqrt{w\prime\sum w}} \text{ subject to } 1\text{'w=1}$$

where μ and ∑ are the estimated mean returns and covariance of group members respectively, and 1 is a vector of ones. (Equivalently, one can solve $Min_w\ w'\sum w$ for a given target return.) The solution is

$$w_{ov} \propto \sum^{-1}(\mu - R_f 1)$$

(normalized so that 1'w_{ov}=1)

Using ovport addresses some issues of simple value-weighting. The optimized portfolio tends to diversify risk by adjusting weights according to covariances: it may underweight very large (or highly volatile) stocks and overweight smaller ones if that improves the portfolio's risk-adjusted return. In effect, weights are determined by risk-return considerations rather than by market cap. As a result, this approach can mitigate the concentration problem noted above. For instance, whereas a cap-weighted portfolio "overweights larger companies" with potentially overpriced securities, the ovport solution will reallocate capital toward stocks that improve the portfolio's Sharpe ratio. Thus, the anomaly-group portfolios obtained via ovport may capture alpha more efficiently while maintaining overall risk control.

In summary, by combining these procedures, our first procedure benchmarks a conventional broad market portfolio, our second procedure is essentially the same but with the application of Shariah filters for Shariah compliancy. Then, our procedures three and four employs machine learning (via LASSO regressions) to refine factor selection, which is mainly aimed to discover portfolios exhibiting anomalies.

5. Results and discussions

In this section, we provide the results of all four of our procedures below. Utilising the prior findings of Abdul Halim & Sukor (2025), we select the anomalous portfolio ME5DY1 to represent our preliminary analysis of the performance all four procedures. We present Table 1 below consisting of total cumulative returns, standard deviation of returns and Sharpe ratio of all four procedures for the entire duration in which, data is available for the anomalous portfolio ME5DY1.

Table 1 – Cumulative returns in decimals, standard deviation and Sharpe ratio of all four procedures. 2000-2023, 23 yearly observations.

Procedures	**Cumulative Returns**	**Standard Deviation**	**Sharpe Ratio**
S&P 500	0.0044	0.0112	0.3905
S&PDJ Islamic Market US	0.0064	0.0050	1.2868
Anomaly based VW Portfolio	0.0046	0.0007	6.4404
Anomaly based Portfolio with *ovport*	0.0529	0.0102	5.1778

Table 1 presents the cumulative returns, standard deviations, and Sharpe ratios for the four portfolio procedures over the 2000–2023 period. The results demonstrate a clear performance hierarchy. The anomaly-based portfolios—particularly those optimized using STATA's ovport function—deliver the strongest performance, with the ovport-based strategy achieving the highest cumulative excess return (0.0529) and a Sharpe ratio of 5.18. Similarly, the anomaly-based value-weighted portfolio also exhibits an exceptionally high Sharpe ratio (6.44), albeit with an unusually low standard deviation (0.0007), which may indicate overly stable return patterns and warrants scrutiny for potential overfitting. In contrast, the conventional S&P 500 portfolio, while yielding a respectable cumulative return (0.0044), reflects a much lower Sharpe ratio (0.39), driven by its higher volatility (0.0112). Notably, the Shariah-compliant S&P Dow Jones Islamic Market (S&PDJIM) U.S. Index delivers both higher excess returns (0.0064) and a Sharpe ratio (1.29) superior to that of the S&P 500, consistent with findings that Islamic indices often exhibit lower volatility due to sectoral exclusions and reduced leverage. Overall, the table supports the proposition that anomaly-oriented strategies—especially when combined with variance optimization—can significantly enhance portfolio efficiency. Nonetheless, the unusually strong performance of the anomaly-based strategies highlights the importance of validating their robustness through out-of-sample testing and rolling performance windows.

To address the latter issue, we present the yearly decimal returns for each procedure in Figure 1 below.

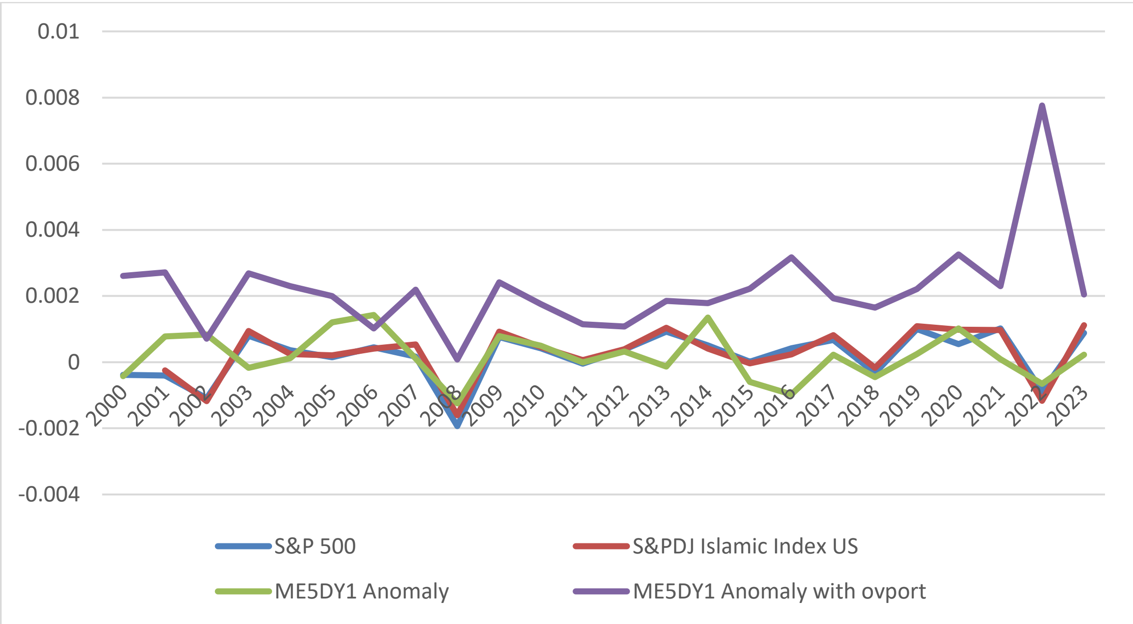

Figure 1 – Yearly decimal returns of all four procedures using ME5DY1 anomalous portfolio.

Figure 1 illustrates the annual return performance of all four portfolio procedures from 2000 to 2023. The results reveal consistent outperformance by the anomaly-based portfolios, particularly the ME5DY1 optimized using the ovport function. These portfolios demonstrate strong positive returns in most years, often exceeding those of both the conventional S&P 500 and the S&P Dow Jones Islamic Market (S&PDJIM) U.S. Index. This pattern suggests that anomaly-based portfolio construction—especially when combined with variance optimization—effectively captures persistent pricing inefficiencies, resulting in superior annual returns.

The anomaly-based strategies also appear to exhibit greater resilience during market downturns. During major crises such as the 2008 Global Financial Crisis and the 2020 COVID-19 shock, the drawdowns observed in the optimized anomaly portfolios are visibly less severe than those of the S&P 500. This relative stability suggests that these strategies may offer some degree of downside protection, possibly due to their selective exposure to firm-specific characteristics associated with higher expected returns.

Meanwhile, the Shariah-compliant S&PDJIM U.S. Index generally tracks the performance of the S&P 500, but with slightly better returns in several years and reduced losses in others. This is consistent with the known defensive characteristics of Islamic equity screens, which typically exclude high-leverage and speculative sectors. Overall, Figure 1 reinforces the idea that anomaly-based, Shariah-compliant portfolios provide a compelling alternative to traditional diversification strategies, delivering both higher returns and more stable performance across different market environments.

So far, a case can be made that anomalous portfolios outperform conventional diversification strategies. Would these results hold for other anomalous portfolios as well? This is particularly important as our proposed LASSA regressions procedure would automate the discovery of anomalous portfolios, and their consistent outperformance is key. We present the performance of another alternative anomalous portfolio, ME5AD5, in Abdul Halim & Sukor (2025), below.

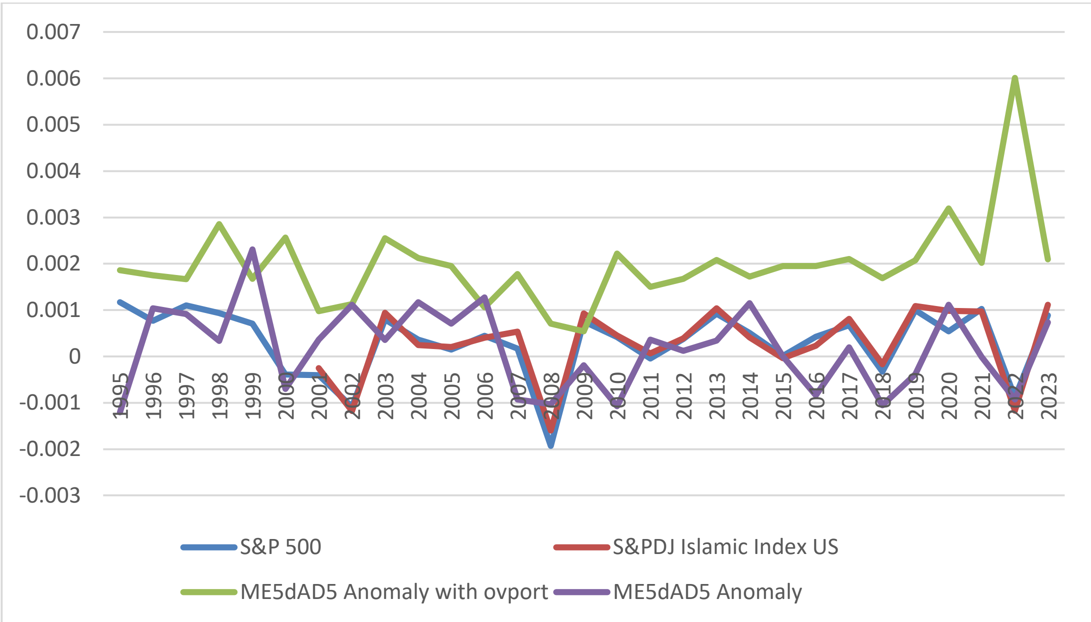

Figure 2 – Yearly decimal returns of all four procedures using ME5dAD5 anomalous portfolio.

Figure 2 illustrates the annual returns of the four portfolio procedures, each constructed using the ME5dAD5 anomaly screening method over the 2000–2023 period. The figure reveals that anomaly-based portfolios—particularly the one optimized via ovport—frequently deliver higher returns relative to both the S&P 500 and the S&P DJIM Islamic Index across multiple years as seen before. The enhanced performance is most evident during stable and rising market periods, suggesting that the ME5dAD5 anomaly criteria successfully capture stocks with superior return potential.

The anomaly-based value-weighted strategy also shows consistently strong returns, although slightly below the ovport-based portfolio. These findings indicate that even without advanced optimization, a carefully selected anomaly-filtered universe of Shariah-compliant stocks can yield significant performance benefits over traditional market benchmarks.

In contrast, the S&P 500 and Islamic index portfolios track the broader market more closely, showing higher variability and less consistent performance. However, during certain downturns, the Islamic index demonstrates relatively better resilience, consistent with previous findings on its defensive characteristics.

Overall, Figure 2 reinforces the central proposition of this paper—that anomaly-based portfolio formation, even within a Shariah-compliant framework, can generate superior annual returns compared to conventional and passively diversified benchmarks.

Clearly, the anomaly-oriented strategies show in-sample evidence of outperformance vis-à-vis the conventional and SC diversified portfolio strategies. At the same time, it is also clear that STATA's mvport significantly increase portfolio performance by allowing for ML to decide the stock's weights. However, are all anomaly-oriented portfolio equal? Will the intensity of anomalies affect the optimized portfolio performance? When using our proposed LASSO regressions, some of the discovered alphas may vary in intensity, which is measured by the absolute value of it's t-statistics. Would a bigger t-statistics anomalous portfolio always outperform a smaller t-statistics anomalous portfolio? We present Figures 3 and 4 below to answer these questions.

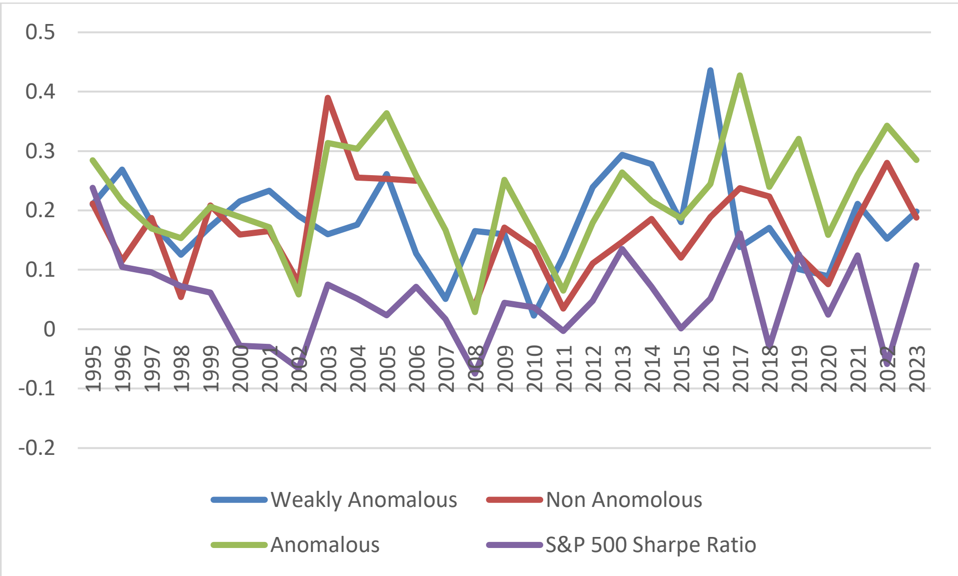

Figure 3 – Yearly Sharpe ratios of the selected Dividend Yield Anomaly Portfolios.

Figure 3 above shows the annual Sharpe ratios of the S&P500, the anomalous portfolio (ME5DY1, t-statistics more than 2), the non-anomalous portfolio (ME2DY5, t-statistics less than 1.6) and the weakly anomalous portfolio (ME3DY1, t-statistics between 1.8 to 1.6). Figure 3 shows that the performance of each portfolio shows some relation to the intensity of the anomaly. It can be clearly seen that the anomalous portfolio and weakly-anomalous portfolio tend to perform better than the Non-anomalous portfolio and the simple market portfolio. In fact, both the anomalous and weakly anomalous portfolio almost always perform better than the market portfolio.

At the same time, we find that the anomalous and weakly anomalous portfolios tend to weather crises period better. Whereas the market portfolio tends to have negative Sharpe ratio during the 2001 dotcom Bubble burst and the 2008 Global Financial Crisis, the anomalous and weakly anomalous portfolios still retain positive Sharpe ratio. In fact, in 2022, all anomalous portfolios show decidedly positive Sharpe ratios, at a time when the market portfolio show negative Sharpe ratio. Perhaps these results remain only for the dividend yield anomaly? We explore this question in Figure 4 below:

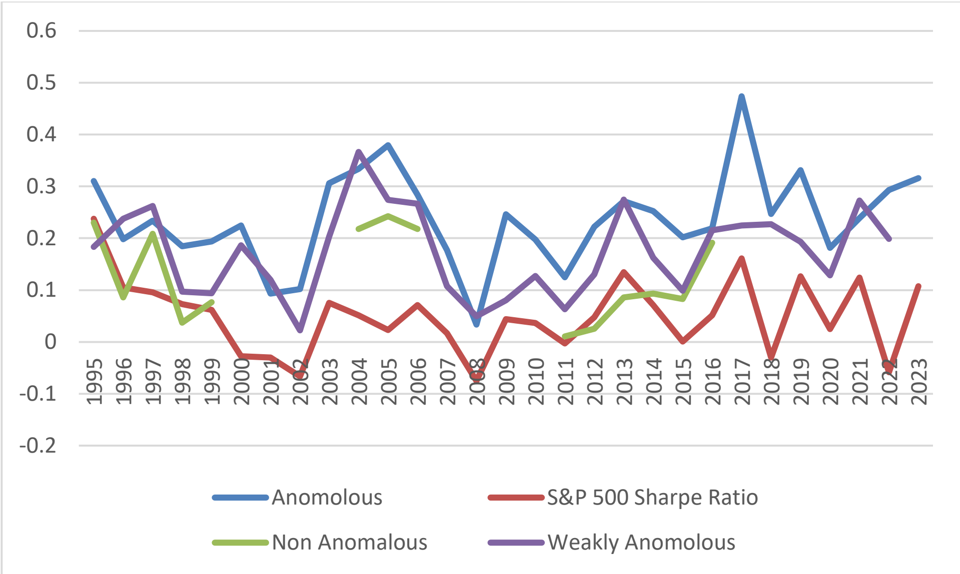

Figure 4 – Yearly Sharpe ratios of the selected change in Actual Dividend Anomaly Portfolios.

Figure 4 above repeats the same exercise in Figure 3 previously, except by changing the dividend yield anomaly with the change in actual dividend anomaly found in Abdul Halim & Sukor (2025). Note that some missing data exists for the non-anomalous portfolio in this anomaly type.

Essentially the same results persist in Figure 4 as seen in Figure 3 before. That is, anomalous portfolios (strong or weak) always outperform the market portfolio. Anomalous portfolios also almost always outperform non-anomalous portfolios. Our findings suggest that there may be a positive relationship between the absolute value of the t-statistics of the anomaly and the Sharpe ratio of the anomaly portfolios. A higher t-statistics should lead to higher Sharpe ratio.

6. Robustness

To evaluate the robustness and out-of-sample performance of our anomaly-oriented, machine learning–assisted portfolio construction procedures, we extend the analysis by collecting return data for the year 2024—beyond the original estimation window, which ended in December 2023. These additional results, presented in Table 2, follow the same structure as Table 1 and allow us to assess whether the strategies maintain their predictive and performance advantages outside the initial sample period.

Table 2 – Cumulative returns in decimals, standard deviation and Sharpe ratio of all four procedures, for the year 2024

Procedures	**Cumulative Returns**	**Standard Deviation**	**Sharpe Ratio**
S&P 500	0.000878	0.007994	0.1098
S&PDJ Islamic Market US	0.000859	0.009407	0.0913
Anomaly based Portfolio with *ovport*	0.001364	0.006856	0.1989

Table 2 presents the out-of-sample performance of some of our procedures for the year 2024, providing a critical test of the robustness of the machine learning–assisted anomaly-based approaches introduced earlier in the study. The anomaly-based portfolio optimized using STATA's ovport function continues to outperform traditional benchmarks, achieving the highest Sharpe ratio (0.1989) alongside the highest cumulative return and the lowest standard deviation. In contrast, both the conventional S&P 500 and the S&P Dow Jones Islamic Market (S&PDJIM) U.S. Index portfolios exhibit lower Sharpe ratios (0.1098 and 0.0913, respectively), indicating weaker risk-adjusted returns. These results suggest that anomaly-oriented strategies retain predictive power and continue to offer superior performance even under new market conditions, reinforcing their practical applicability for portfolio construction.

7. Conclusions and implications

This paper demonstrates that integrating machine learning—specifically LASSO regressions—into portfolio optimization can significantly enhance the performance of Shariah-compliant (SC) equity investments in the United States. By identifying asset pricing anomalies within SC stocks and employing these insights through both value-weighted and mean-variance optimization strategies, we find strong in-sample evidence of outperformance relative to conventional market portfolios. The anomaly-based portfolios, particularly those optimized using STATA's ovport function, consistently deliver higher Sharpe ratios and show resilience during periods of market turbulence. Our findings suggest that anomaly intensity, measured via the t-statistics of alpha, may a positive relationship with anomalous portfolio outperformance.

From a policy perspective, these results have important implications for Islamic finance regulators, fund managers, and institutional investors. Firstly, Islamic financial institutions and sovereign wealth funds might benefit from adopting machine learning–assisted anomaly detection in their equity selection processes, enabling them to align ethical investment principles with competitive financial performance. More broadly, these findings support the idea that responsible investing—when combined with data-driven techniques—can deliver both moral and market value.

Nonetheless, this study is not without limitations. Despite some out-of-sample validation, the results rely heavily on in-sample analysis, and the high Sharpe ratios raise the possibility of overfitting. Future work should explore rolling window or out-of-sample validations to ensure robustness. Additionally, while LASSO offers interpretability, it assumes linear relationships and may overlook nonlinear patterns that other ML methods could capture. Expanding this framework to include more advanced models (e.g., tree-based, ensemble, or neural networks) and testing it across global SC markets could offer valuable generalizations. Specifically, this anomaly-oriented ML framework is data-driven and not reliant on unique US market microstructures. Therefore, it holds significant potential for generalizability to other key Islamic finance hubs, such as the GCC region, Malaysia, and Indonesia, where similar liquidity and data availability standards are increasingly being met. Future research should also investigate whether ML-assisted anomaly strategies can incorporate real-world constraints like transaction costs, liquidity limits, or ESG compliance thresholds, to enhance practical applicability.

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