

## Integrating Six Sigma Graduated Standards with Joint Confidence Blocks for Multi- Characteristic Process Capability Analysis

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**Abstract:** Traditional Process Capability Analysis Charts for  $C_{pm}$  (PCAC $_{pm}$ ) incorporating Joint Confidence Blocks (JCB) successfully address sampling uncertainty but face implementation barriers when fixed capability standards prove excessively stringent for practical manufacturing contexts. This study develops a Six Sigma-based PCAC $_{pm}$  framework integrating graduated evaluation standards ( $3\sigma$ ,  $4\sigma$ ,  $5\sigma$ ,  $6\sigma$ ) with JCB methodology to simultaneously preserve statistical rigor while enhancing industrial implementability. The framework employs systematic six-step procedures enabling manufacturers to select capability thresholds appropriate to organizational quality maturity rather than imposing uniform fixed standards. Empirical validation at a precision tool manufacturer examining socket storage trays with three dimensional characteristics demonstrated the methodology's effectiveness: under  $5\sigma$  standards ( $C_0=1.034$ ), width achieved borderline qualification while length and height required targeted improvement interventions addressing precision and combined accuracy-precision deficiencies respectively. Comparative analysis revealed that traditional "capable" standards ( $c=1.33$ ) would classify all characteristics as non-qualified, potentially discouraging methodology adoption, whereas graduated Six Sigma standards provided realistic yet challenging targets supporting progressive quality improvement. The framework's theoretical contribution lies in decoupling statistical technique from evaluation criteria, creating versatile assessment tools maintaining analytical rigor across varying capability thresholds while addressing practical adoption barriers through flexible graduated standards.

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**Keywords:** Process capability analysis; Six Sigma standards; Joint confidence blocks; Multi-characteristic products; Quality control; Sampling error.

## 1. Introduction

Process Capability Indices (PCIs) serve as fundamental metrics in statistical quality control, quantifying manufacturing process conformance to specifications (Kane, 1986; Chan et al., 1988). For multi-characteristic products, the Process Capability Analysis Chart for  $C_{pm}$  (PCAC<sub>pm</sub>) enables simultaneous evaluation across multiple dimensional or performance attributes (Chen et al., 2001; Hsu et al., 2015).

Traditional PCAC<sub>pm</sub> implementations encounter two critical challenges. First, point estimation approaches fail to account for sampling variability inherent in practical quality assessment (Chen et al., 2009). Recent advances incorporating Joint Confidence Blocks (JCB) successfully address this limitation by substituting point estimates with confidence intervals (Hsu, 2017). However, a second challenge persists: traditional capability standards ( $c=1.33$  for "capable" classification) often prove excessively stringent for real-world manufacturing conditions, creating adoption barriers despite the methods' statistical rigor.

This implementation barrier manifested at O Company, a precision tool storage manufacturer, when adopting JCB-enhanced PCAC<sub>pm</sub>. Multiple quality characteristics meeting customer requirements and functional specifications consistently failed stringent capability thresholds, creating a critical dilemma: maintain theoretically rigorous but impractical standards, or abandon advanced methodology altogether. This experience exemplifies a broader challenge—manufacturers require assessment tools simultaneously rigorous enough to provide reliable guidance yet flexible enough to accommodate realistic production capabilities.

This study develops a Six Sigma-based PCAC<sub>pm</sub> framework preserving JCB's uncertainty quantification while incorporating graduated evaluation standards suited to diverse manufacturing contexts. Two key objectives guide this research:

1. Develop graduated capability assessment criteria across multiple sigma levels, enabling manufacturers to select thresholds appropriate to their quality maturity stages rather than imposing uniform fixed standards.
2. Integrate Six Sigma graduated standards with JCB methodology to create a comprehensive framework simultaneously addressing sampling uncertainty through interval estimation and practical feasibility through flexible evaluation criteria.

By decoupling statistical technique from evaluation benchmarks, this framework provides manufacturers with versatile quality assessment tools balancing analytical sophistication with operational practicality, enabling progressive quality improvement pathways from current capability levels toward excellence.

## 2. Process Capability Analysis Chart $C_{pm}$ (PCAC<sub>pm</sub>)

Process Capability Indices (PCIs) serve as essential metrics for manufacturing quality assessment. The foundational index  $C_p$ , developed for normally distributed characteristics, establishes relationships between process variability and specification boundaries (Kane, 1986):

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma} \quad (1)$$

where USL and LSL denote specification limits,  $\sigma$  represents process standard deviation, and  $d = \frac{USL - LSL}{2}$

However,  $C_p$  fails to account for process centering (Kane, 1986). The index  $C_{pk}$  incorporates mean location  $\mu$  (Kane, 1986; Pearn & Chen, 1997):

$$C_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} = \frac{d - |\mu - m|}{3\sigma} \quad (2)$$

where  $m = \frac{USL + LSL}{2}$  represents the specification midpoint.

Both  $C_p$  and  $C_{pk}$  demonstrate independence from target value  $T$ . The index  $C_{pm}$  addresses this limitation by incorporating target deviation (Chan et al., 1988):

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} \quad (3)$$

Adopting accuracy  $A = \frac{\mu - T}{d}$  and precision  $P = \frac{\sigma}{d}$  (Chen et al., 2001; Hsu et al., 2015), equation (3) becomes:

$$C_{pm} = \frac{1}{3\sqrt{P^2 + A^2}} \quad (4)$$

For products with multiple quality characteristics, overall capability consistently falls below individual characteristic capabilities (Chen et al., 2006). Following established methodology (Ouyang et al., 2013), the minimum critical value  $C_0$  is:

$$C_0 = \frac{1}{3} \Phi^{-1} \left( \frac{[2\Phi(3C) - 1]^{1/t} + 1}{2} \right) \quad (5)$$

where  $c$  represents the integrated PCI,  $t$  denotes the number of quality characteristics, and  $\Phi$  is the standard normal cumulative distribution function (Pearn & Chen, 1997).

Traditional applications employ fixed capability levels (inadequate:  $c=1.0$ , capable:  $c=1.33$ , satisfactory:  $c=1.5$ , superior:  $c=2.0$ ) (Montgomery, 2009). While effective in many contexts, these uniform standards may prove either insufficiently demanding or unrealistically stringent depending on industry context, product complexity, or organizational quality maturity. This inflexibility creates adoption barriers, particularly for organizations transitioning toward advanced quality management systems (Spiring et al., 2003).

Six Sigma methodology provides graduated quality assessment recognizing varying capability levels across manufacturing contexts (Harry & Schroeder, 2000). Rather than fixed thresholds, Six Sigma establishes a spectrum of quality levels based on process standard deviations within specification limits, representing progressively stringent defect rate requirements:  $3\sigma$  (66,807 DPMO),  $4\sigma$  (6,210 DPMO),  $5\sigma$  (233 DPMO), and  $6\sigma$  (3.4 DPMO) (Pyzdek & Keller, 2014). This graduated structure enables organizations to select realistic capability targets aligned with current performance while establishing clear pathways toward higher quality levels.

For multi-characteristic products, equation (5) applies using Six Sigma levels to calculate corresponding  $C_0$  values. Table 1 presents  $C_0$  values across multiple quality characteristics ( $t$ ) under four Six Sigma standards, extending the traditional framework from fixed thresholds to graduated assessment criteria that accommodate diverse organizational contexts and improvement trajectories.

Table 1  $C_0$  Values for Six Sigma Standards

t	$3\sigma$ ( $C=0.55$ )	$4\sigma$ ( $C=0.74$ )	$5\sigma$ ( $C=0.92$ )	$6\sigma$ ( $C=1.11$ )
1	0.55	0.74	0.92	1.11
2	0.651	0.825	0.993	1.173
3	0.706	0.872	1.034	1.208
4	0.744	0.904	1.062	1.233
5	0.772	0.929	1.083	1.251
6	0.794	0.948	1.100	1.267
7	0.813	0.964	1.114	1.279

Setting  $C_{pm} = C_0$  establishes the capability boundary relationship (Hsu et al., 2015):

$$C_{pm} = C_0 = \frac{1}{3\sqrt{P^2 + A^2}} \quad (6)$$

$$\text{Therefore: } P^2 + A^2 = \frac{1}{(3C_0)^2}$$

This defines a semicircular boundary in A-P coordinate space, where region  $P^2 + A^2 \leq \frac{1}{(3C_0)^2}$  represents qualified capability. Figure 1 illustrates this framework under Six Sigma standards, displaying multiple graduated boundaries corresponding to different sigma levels. The concentric semicircular boundaries create distinct capability zones: characteristics falling within the innermost ( $6\sigma$ ) boundary demonstrate highest capability, while those between boundaries indicate intermediate performance levels, providing manufacturers with clear visual differentiation of quality achievement stages.

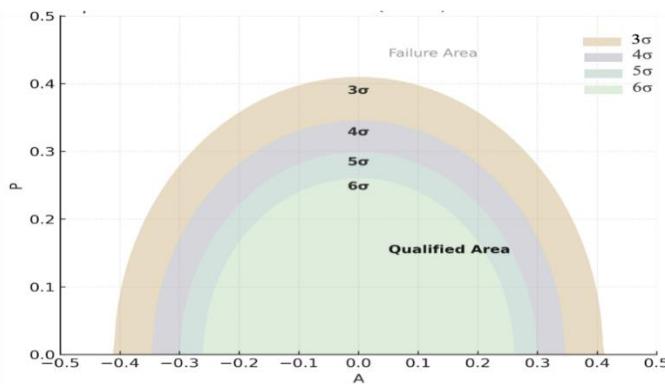


Figure 1 PCAC<sub>pm</sub> Framework under Six Sigma Standards - Multiple Graduated Boundaries ( $t=7$ )

By adopting graduated Six Sigma standards rather than fixed traditional thresholds, manufacturers gain flexibility to establish realistic capability targets matching current organizational maturity while maintaining clear progression pathways toward excellence. This adaptability addresses practical implementation barriers while preserving PCAC<sub>pm</sub> analytical rigor.

Traditional PCAC<sub>pm</sub> implementations utilize point estimates that fail to capture sampling uncertainty (Chen et al., 2009). When sample data rather than population parameters inform assessments, point estimates provide incomplete reliability information.

This limitation persists regardless of capability standard employed—sampling error requires explicit treatment through interval estimation (Castagliola & Castellanos, 2005).

Joint Confidence Blocks (JCB) provide rigorous quantification of sampling uncertainty (Hsu, 2017). According to Boole's inequality, the 100(1- $\alpha$ )% JCB for indices A and P is:

$$P \left\{ \begin{array}{l} \hat{A} - t_{\alpha/2}(n-1) \frac{\hat{P}}{\sqrt{n}} \leq A \leq \hat{A} + t_{\alpha/2}(n-1) \frac{\hat{P}}{\sqrt{n}}, \\ \left[ \frac{(n-1)\hat{P}^2}{\chi_{1-\alpha/2}^2(n-1)} \right]^{1/2} \leq P \leq \left[ \frac{(n-1)\hat{P}^2}{\chi_{\alpha/2}^2(n-1)} \right]^{1/2} \end{array} \right\} \geq 1 - \alpha_1 - \alpha_2 \quad (7)$$

Where  $t_{\alpha/2}$  and  $\chi_{\alpha/2}^2$  represent t-distribution and  $\chi^2$  distribution percentiles with (n-1) degrees of freedom,  $A = \frac{\bar{x}-T}{d}$ , and  $\hat{P} = \frac{s}{d}$ .

Setting  $\alpha_1 = \alpha_2 = \alpha/2$ , the Cartesian product  $S(X)$  of the 100(1- $\alpha$ )% JCB becomes (Hsu, 2017):

$$S(X) = [A_1, A_2] \times [P_1, P_2] \quad (8)$$

Where:

$$A_1 = \hat{A} - t_{\alpha/4}(n-1) \cdot \frac{\hat{P}}{\sqrt{n}}, \quad A_2 = \hat{A} + t_{\alpha/4}(n-1) \cdot \frac{\hat{P}}{\sqrt{n}}$$

$$P_1 = \left[ \frac{(n-1)\hat{P}^2}{\chi_{1-\alpha/4}^2(n-1)} \right]^{1/2}, \quad P_2 = \left[ \frac{(n-1)\hat{P}^2}{\chi_{\alpha/4}^2(n-1)} \right]^{1/2}$$

These interval estimates replace point assessments with rectangular confidence regions in A-P space, explicitly quantifying sampling uncertainty. Critically, JCB methodology remains independent of capability standards—it addresses sampling error rather than evaluation criteria selection.

The preceding sections establish two complementary enhancements: Six Sigma graduated standards address evaluation threshold selection, providing multiple capability levels adaptable to diverse organizational contexts and improvement stages; JCB methodology addresses sampling uncertainty, transforming point estimates into interval assessments with explicit confidence levels.

These enhancements are independent yet synergistic. Six Sigma standards modify the capability boundaries ( $C_0$  values in equation 6), creating multiple concentric semicircular thresholds rather than a single fixed boundary, while JCB methodology modifies characteristic representation from points to confidence rectangles. Together, they create an integrated framework simultaneously addressing practical feasibility through graduated standards and statistical rigor through interval estimation.

### 3. Implementation Methodology

The implementation methodology integrates Six Sigma graduated standards for flexible capability thresholds with JCB techniques for sampling uncertainty quantification. Unlike traditional approaches imposing uniform evaluation criteria, this framework enables organizations to select appropriate sigma levels based on quality maturity and operational requirements while maintaining rigorous interval-based assessment.

The methodology transforms traditional point-based capability assessment into a comprehensive interval-based framework. Rather than representing each quality characteristic as a single point in the A-P coordinate space, the approach constructs confidence rectangles that explicitly quantify sampling uncertainty. Combined with graduated capability boundaries, this transformation enables nuanced assessment distinguishing between characteristics meeting different sigma standards, supporting progressive quality improvement strategies.

The Six Sigma-based JCB-enhanced PCAC<sub>pm</sub> methodology follows a systematic six-step procedure:

**Step 1: Six Sigma Parameter Determination**

Identify the total number of quality characteristics ( $t$ ) in the product under evaluation. Select the appropriate Six Sigma capability level ( $3\sigma$ ,  $4\sigma$ ,  $5\sigma$ , or  $6\sigma$ ) based on organizational quality requirements. Determine the critical value  $C_0$  corresponding to the selected sigma level and characteristic count  $t$  using Table 1. This  $C_0$  value establishes the capability boundary for subsequent assessment.

**Step 2: PCAC<sub>pm</sub> Framework Construction**

Construct the analytical chart using the fundamental boundary relationship:

$$P^2 + A^2 = \frac{1}{(3C_0)^2},$$

This equation defines a semicircular boundary in the A-P coordinate space with radius  $r = \frac{1}{3C_0}$ . The qualified region lies within the semicircular boundary  $P^2 + A^2 = \frac{1}{(3C_0)^2}$ , while the region beyond indicates characteristics requiring process improvement. Organizations implementing progressive quality strategies may display multiple sigma boundaries simultaneously to assess achievement relative to different standards.

**Step 3: Data Collection and Confidence Level Setting**

Establish the confidence level ( $1-\alpha$ ) for JCB construction, typically 95% ( $\alpha = 0.05$ ) for manufacturing quality assessment. Determine appropriate sample size ( $n$ ) for each quality characteristic, balancing statistical adequacy with operational feasibility. Typical sample sizes range between 20 and 50 units.

Implement systematic sampling procedures ensuring representative data collection for each quality characteristic. For each characteristic, collect sample data and compute sample mean ( $\bar{x}$ ) and sample standard deviation ( $s$ ).

**Step 4: JCB Interval Computation**

For each quality characteristic, calculate point estimates:

$$A = \frac{\bar{x} - T}{d}, P = \frac{s}{d}$$

$$\text{Where } d = \frac{USL - LSL}{2}$$

Compute the  $100(1-\alpha)\%$  Joint Confidence Blocks. Setting  $\alpha_1 = \alpha_2 = \alpha/2$ , the JCB boundaries are:

$$A_1 = \hat{A} - t_{\alpha/4}(n-1) \cdot \frac{\hat{P}}{\sqrt{n}}, A_2 = \hat{A} + t_{\alpha/4}(n-1) \cdot \frac{\hat{P}}{\sqrt{n}}$$

$$\text{Precision interval: } P_1 = \left[ \frac{(n-1)\hat{P}^2}{X_{1-\alpha/4}^2(n-1)} \right]^{1/2}, P_2 = \left[ \frac{(n-1)\hat{P}^2}{X_{\alpha/4}^2(n-1)} \right]^{1/2}$$

The Cartesian product  $[A_1, A_2] \times [P_1, P_2]$  defines a rectangular confidence region for each quality characteristic, explicitly quantifying sampling uncertainty.

**Step 5: JCB Visualization and Capability Assessment**

Plot the JCB rectangles for all quality characteristics on the PCAC<sub>pm</sub> chart. Assess qualification status based on JCB rectangle position relative to the capability boundary:

- (1) Qualified: The entire JCB rectangle falls within the capability boundary. These characteristics meet the established sigma standard with statistical confidence.
- (2) Non-qualified: Any portion of the JCB rectangle extends beyond the capability boundary. These characteristics require process improvement attention.

#### Step 6: Decision Making and Improvement Prioritization

Identify characteristics requiring process improvement and establish enhancement priorities based on capability assessment results. Analyze whether deficiency stems from accuracy issues, precision issues, or both to enable targeted improvement interventions. Allocate quality improvement resources based on capability assessment outcomes, with non-qualified characteristics receiving focused attention proportional to deficiency severity and characteristic criticality.

#### 4. Case Study

O Company, established in 1985, specializes in precision tool storage solutions with particular expertise in socket organization systems. As a leading manufacturer in Taiwan's tool accessories industry, the company supplies products to major international tool brands and maintains strong quality management capabilities developed through nearly four decades of manufacturing experience.

This study examines the STR1432 model from O Company's 3 Row Socket Tray series. The STR1432, designed for 1/2" drive socket storage with dimensions 467mm × 150mm × 100mm, represents a precision-molded product requiring stringent dimensional control for proper functionality. As a professional-grade storage solution targeting workshop tool cabinets and industrial applications, the product exemplifies multi-characteristic quality assessment challenges common in precision manufacturing.

The STR1432 socket storage tray encompasses three critical dimensional characteristics essential for product functionality and customer satisfaction. Table 2 presents the quality specifications for these characteristics, determined according to general machining tolerance standards appropriate for precision molded components.

*Table 2 STR1432 Quality Characteristics and Specifications*

Quality Characteristic	Specification (mm)	Target (T)	Quality Impact
Length (x <sub>1</sub> )	467 ± 0.8	467	Cabinet drawer fitment
Width (x <sub>2</sub> )	150 ± 0.5	150	Socket arrangement spacing
Height (x <sub>3</sub> )	100 ± 0.3	100	Vertical clearance and stacking

Following the six-step methodology established in previous section, the implementation process for STR1432 quality assessment proceeds systematically:

#### Step 1: Six Sigma Parameter Determination

The STR1432 socket tray contains three quality characteristics (t=3). Based on organizational quality objectives and industry standards for precision molded components, the 5 $\sigma$  capability level (c=0.92) was selected. This standard represents mature manufacturing capability appropriate for professional-grade tool accessories, balancing quality requirements with operational feasibility. Using Table 1, the critical value C<sub>0</sub>=1.034

is determined for  $t=3$  and  $5\sigma$  standard. This establishes the capability boundary for subsequent assessment.

#### Step 2: PCAC<sub>pm</sub> Framework Construction

The analytical chart boundary is constructed using the relationship  $P^2+A^2=1/(3C_0)^2=1/(3\times 1.034)^2\approx 0.1039$ , creating a semicircular boundary with radius approximately 0.322.

#### Step 3: Data Collection and Confidence Level Setting

A confidence level of 95% ( $\alpha=0.05$ ) was established for JCB construction. Systematic random sampling with sample size  $n=30$  was implemented for each quality characteristic. Table 3 presents the sample statistics for each quality characteristic.

*Table 3 Sample Statistics for STR1432 Quality Characteristics (n=30)*

Characteristic	$\bar{x}$ (mm)	s (mm)	$\hat{A}$	$\hat{P}$
Length ( $x_1$ )	467.01	0.342	0.0091	0.4270
Width ( $x_2$ )	150.01	0.133	0.0163	0.2655
Height ( $x_3$ )	99.99	0.131	-0.0370	0.4350

#### Step 4: JCB Interval Computation

Joint Confidence Blocks were computed using t-distribution critical value  $t_{0.0125}(29)=2.462$  for accuracy intervals, and chi-square distribution critical values  $\chi^2_{0.0125}(29)=16.047$  and  $\chi^2_{0.9875}(29)=45.722$  for precision intervals. The Cartesian product  $[A_1, A_2] \times [P_1, P_2]$  defines rectangular confidence regions explicitly quantifying sampling uncertainty.

#### Step 5: JCB Visualization and Capability Assessment

Table 4 presents the computed JCB interval estimates for each characteristic.

*Table 4 JCB Interval Estimates for STR1432 Quality Characteristics*

Characteristic	$\hat{A}_1$	$\hat{A}_2$	$\hat{P}_1$	$\hat{P}_2$
Length ( $x_1$ )	-0.1828	0.2010	0.3401	0.5740
Width ( $x_2$ )	-0.1030	0.1356	0.2114	0.3569
Height ( $x_3$ )	-0.2325	0.1585	0.3464	0.5848

Figure 2 displays the  $5\sigma$  capability boundary ( $C_0=1.034$ ) as a semicircular threshold with radius  $r\approx 0.322$  in the A-P coordinate space. Each quality characteristic is represented by its JCB rectangle rather than a single point, with rectangle dimensions explicitly quantifying sampling uncertainty. The hatched patterns indicate non-qualified rectangles extending beyond the capability boundary.

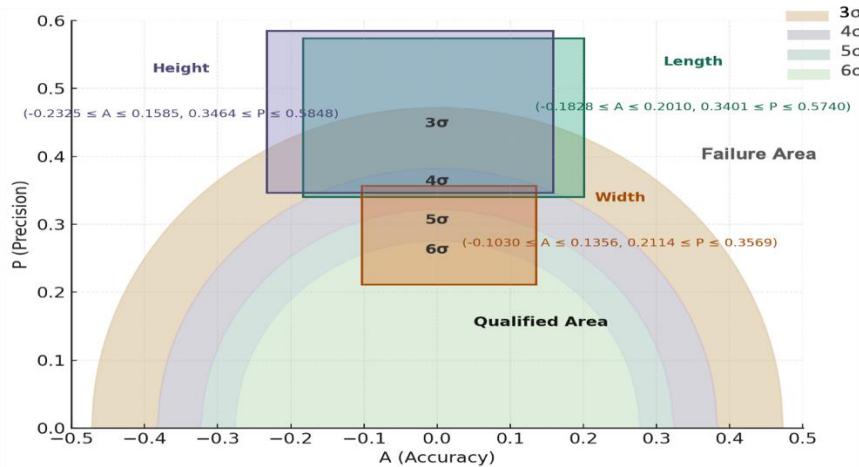


Figure 2 PCAC<sub>pm</sub> Chart with JCB Analysis for STR1432 Quality Characteristics (Based on  $t = 3$  under Six Sigma Standards)

Assessment of JCB rectangle positions relative to the  $5\sigma$  capability boundary ( $P^2+A^2 \leq 0.1039$ ) reveals width ( $x_2$ ) partially spans the boundary indicating borderline qualification, while length ( $x_1$ ) and height ( $x_3$ ) extend entirely beyond the boundary indicating non-qualified status. Length exhibits primarily precision deficiency ( $\hat{P}=0.4270$ ), while height demonstrates combined accuracy ( $\hat{A}=-0.0370$ ) and precision ( $\hat{P}=0.4350$ ) challenges.

#### Step 6: Results Interpretation and Improvement Recommendations

Length ( $x_1$ ) exhibits primarily precision deficiency with excessive variability ( $s=0.342\text{mm}$ ). Recommended improvement strategies include process control enhancement through variation source investigation, equipment maintenance verification, and statistical process control implementation.

Height ( $x_3$ ) demonstrates severe capability deficiency with combined accuracy and precision challenges. This characteristic demands prioritized improvement attention through comprehensive process analysis addressing both centering and variability issues.

Width ( $x_2$ ) approaches  $5\sigma$  capability standards with well-controlled variability ( $s=0.133\text{mm}$ ) and excellent centering. Continued process monitoring is recommended to ensure sustained capability.

Table 5 presents comparative assessment results under different evaluation criteria, illustrating how evaluation standard selection impacts qualification determination and resource allocation decisions.

Table 5 Capability Assessment Under Different Standards

Characteristic	4σ ( $C_0=0.872$ )	5σ ( $C_0=1.034$ )	Traditional $c=1.33$ ( $C_0=1.414$ )
Length ( $x_1$ )	Partially qualified	Non-qualified	Non-qualified
Width ( $x_2$ )	Qualified	Partially qualified	Non-qualified
Height ( $x_3$ )	Non-qualified	Non-qualified	Non-qualified

Under  $4\sigma$  standards ( $C_0=0.872$ , boundary:  $P^2+A^2 \leq 0.174$ ), width achieves full qualification while length shows partial qualification, indicating that the organization

currently operates between  $4\sigma$  and  $5\sigma$  capability levels for dimensional control. The  $5\sigma$  standard selected for this assessment provides appropriate challenge while remaining achievable through focused improvement.

In contrast, applying traditional "capable" standards ( $c=1.33$ ,  $C_o=1.414$ , boundary:  $P^2+A^2 \approx 0.056$ ) would classify all three characteristics as non-qualified, potentially creating the implementation barrier that motivated this research. This comparative analysis validates the Six Sigma-based framework's practical advantage: by enabling appropriate standard selection, the methodology provides realistic yet challenging capability targets supporting progressive quality improvement.

## 5. Discussion

Traditional PCAC<sub>pm</sub> implementations employ fixed capability standards ( $c=1.33$  for "capable" classification) imposing uniform requirements regardless of manufacturing context. This inflexibility creates adoption barriers when processes cannot immediately achieve stringent standards despite producing functionally acceptable products. The O Company case study demonstrates this challenge—under traditional  $c=1.33$  standards ( $C_o=1.414$  for  $t=3$ ), all STR1432 characteristics would classify as non-qualified, potentially discouraging methodology adoption.

The Six Sigma-based framework addresses this limitation by providing graduated evaluation standards enabling realistic yet challenging capability targets. The  $5\sigma$  standard ( $C_o=1.034$ ) applied in the case study provides appropriate challenge while maintaining implementation feasibility, enabling meaningful capability discrimination. This flexibility proves particularly valuable for organizations at varying quality maturity stages, supporting progressive improvement from baseline capability through successive sigma achievements rather than imposing immediately unattainable thresholds.

The graduated standards preserve analytical rigor through consistent JCB-based uncertainty quantification across all sigma levels. By decoupling statistical technique from evaluation criteria, the framework provides versatile assessment tools adaptable to diverse manufacturing contexts while maintaining statistical validity (Pyzdek & Keller, 2014).

The framework offers distinct advantages by simultaneously addressing sampling uncertainty through JCB interval estimation and practical feasibility through graduated Six Sigma standards. The combined approach enables rigorous capability assessment with realistic evaluation criteria, supporting informed resource allocation decisions. Visual representation through PCAC<sub>pm</sub> charts facilitates intuitive capability status communication across organizational levels.

The methodology assumes normally distributed quality characteristics and statistically controlled processes, requiring preliminary data transformation or process stabilization when these conditions are not met. Small sample sizes may produce unreliable wide confidence intervals, and successful implementation requires statistical expertise for proper JCB computation and interpretation.

## 6. Conclusion

Traditional process capability analysis methods employing fixed evaluation standards create implementation barriers when stringent thresholds prove unattainable for certain manufacturing contexts despite functionally acceptable product quality. This study developed a Six Sigma-based JCB-enhanced PCAC<sub>pm</sub> framework addressing dual challenges: sampling uncertainty quantification through interval estimation and practical

feasibility through graduated evaluation standards. The O Company case study demonstrated the framework's effectiveness in conducting rigorous multi-characteristic capability assessment while maintaining realistic evaluation criteria, enabling precise identification of characteristics requiring improvement intervention versus those warranting continued monitoring.

This research extends process capability analysis methodology by integrating graduated Six Sigma standards with JCB-based uncertainty quantification. The theoretical contribution lies in decoupling statistical technique from evaluation criteria, creating versatile assessment tools maintaining analytical rigor across varying capability thresholds. The practical contribution addresses adoption barriers by replacing uniform fixed standards with flexible graduated criteria, enabling organizations to select appropriate sigma levels matching quality maturity while preserving robust interval-based assessment. The framework provides manufacturers with enhanced decision-making tools balancing statistical sophistication with operational practicality.

Manufacturing organizations can leverage the framework for strategic quality planning by establishing progressive capability roadmaps aligned with business objectives. The graduated standards enable documentation of capability advancement from baseline assessments through successive sigma achievements, supporting quality investment justification and competitive differentiation. The interval-based assessment improves resource allocation precision by explicitly quantifying uncertainty, distinguishing characteristics definitively requiring intervention from those warranting monitoring, enabling focused deployment where improvement needs are clearest.

However, successful implementation requires careful consideration of practical challenges. Organizations should anticipate 2-3 months for initial implementation, encompassing personnel training in JCB computation and interval interpretation, data collection infrastructure establishment, and quality management software integration. These resource commitments often trigger change management challenges as quality personnel transition from familiar point estimation approaches to interval-based assessment methodologies, necessitating organizational support for gradual adoption and sustained capability building (Antony et al., 2017).

The framework demonstrates broad applicability across diverse industrial contexts with sector-specific considerations. In electronics manufacturing, where component miniaturization demands tight tolerances across numerous dimensional characteristics, the multi-characteristic assessment capability proves particularly valuable, though higher sigma standards ( $5\sigma$ - $6\sigma$ ) typically apply given industry quality expectations (Goh, 2002). Automotive component manufacturers benefit from the graduated standards approach, as supply chain tier positioning often dictates appropriate capability levels—Tier 1 suppliers to premium brands may target  $5\sigma$ - $6\sigma$  standards while Tier 2 suppliers serving mass-market segments might appropriately implement  $4\sigma$ - $5\sigma$  thresholds (Kumar et al., 2006). These examples underscore the framework's flexibility advantage: organizations can tailor sigma level selection to industry context and organizational quality maturity rather than adhering to uniform rigid standards.

Future research should extend the methodology to accommodate non-normal distributions through robust statistical techniques, broadening applicability across diverse manufacturing contexts where normality assumptions may not hold. Further empirical validation across diverse industries and product types would strengthen generalizability claims and identify sector-specific implementation considerations, providing comprehensive evidence of the framework's versatility in varying manufacturing environments.

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